

# Complete Hamiltonian analysis of cosmological perturbations at all orders

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**Abstract.** In this work, we present a consistent Hamiltonian analysis of cosmological perturbations at all orders. To make the procedure transparent, we consider a simple model and resolve the ‘gauge-fixing’ issues and extend the analysis to scalar field models and show that our approach can be applied to any order of perturbation for any first order derivative fields. In the case of Galilean scalar fields, our procedure can extract constrained relations at all orders in perturbations leading to the fact that there is no extra degrees of freedom due to the presence of higher time derivatives of the field in the Lagrangian. We compare and contrast our approach to the Lagrangian approach (Chen et al [2006]) for extracting higher order correlations and show that our approach is efficient and robust and can be applied to any model of gravity and matter fields without invoking slow-roll approximation.

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## 1 Introduction

Linear order cosmological perturbation theory [1–7] has been highly successful at describing the CMB anisotropies [8]. It also helps to describe the seed Gaussian density perturbations during inflation and the formation of large scale structures [9, 10]. While linear order perturbations are fairly understood, there are several open issues in applying theory beyond linear order both early and late time universe [5, 6, 9, 11–14]. In the last decade, the possibility of observing primordial non-Gaussianity in CMB [8] and potentially ruling out inflationary models [5, 13, 15, 16] has led to a lot of interest in higher order perturbations.

Currently, there are two formalism in the literature to study gauge invariant cosmological perturbations — Hamiltonian [2] and Lagrangian formulation [1, 3, 17–37]. In the Lagrangian formulation, one needs to either perturb Einstein’s equations or vary the perturbed Lagrangian to obtain perturbed equations of motion. In the Hamiltonian formulation, gauge-invariant first order perturbed equations are obtained in terms of field variables and their conjugate momenta.

In the context of early universe, evaluation of  $n$ -point correlation functions of the effective (scalar) field requires the quantum Hamiltonian of this effective field. For matter fields containing first derivative in time, it is straightforward to obtain the Hamiltonian by performing Legendre transformation. Equations of motion of such fields contain upto second derivative of the field variables which can be linearized as two independent coupled first order differential equations (Hamilton’s equations) — one corresponding to the time evolution of the field and other corresponding to the time evolution of the momentum. This indicates that the phase space is two-dimensional.

However, for higher derivative field theories, the Hamiltonian structure and the associated degrees of freedom is not straightforward. For any higher (more than one) derivative theories, the equations of motion have upto two times the highest order derivative of the field. For example, fields with second order time derivatives, the equation motion contain upto fourth order derivatives of the field. So, if we linearize the equation, we obtain four independent coupled first order differential equations which indirectly imply that, the phase-space is four dimensional and can be mapped to Hamiltonian of two fields. However, the mapped Hamiltonian has unbounded negative energy leading to Ostrogradsky’s instability [38, 39]. This implies that extra degrees of freedom (named as ghost) for a higher derivative Lagrangian causes the instability and hence, in general, quantizing the Hamiltonian is not possible.

On the contrary, Galilean scalar field [40–42] is a special higher derivative field which leads to second derivative equations of motion, implying that the the phase space contains one independent variable and one corresponding momentum, although, multiple variable as well as momenta may appear in the Hamiltonian. Also the absence of extra degrees of freedom leads to the fact that, Hamiltonian of Galilean field is bounded can be quantized.

The main aim of the work is to write the effective Hamiltonian of the generalized (Galilean) scalar field coupled to gravity at all orders in perturbations. Hamiltonian approach to the cosmological perturbations has not been extensively studied in the literature. Langlois [2] showed that, equations of motion of canonical scalar field can be obtained in a gauge-invariant single variable form at first order perturbation. However, Langlois’ approach can not be extended to include higher order due to fact that the approach requires construction of gauge-invariant conjugate momentum. Another aim of this work is to extend Langlois’ analysis to higher orders. It is necessary to extend Langlois’ method to higher order for the following reasons. First, to calculate higher order correlation functions, currently several

approximations are employed to convert effective Lagrangian to Hamiltonian [43]. Our aim is to provide a simple, yet robust procedure to calculate Hamiltonian for arbitrary field(s) at all orders in cosmological perturbations. Second, as mentioned earlier, we do not have a procedure to perform Hamiltonian analysis for the Galilean fields. The procedure we adopt here can be extended to Galilean fields; we explicitly show this in this work. Deffayet et al [44] gave a mechanism to deal with Galilean theory in the context of General relativity. Third, the procedure can be used to include quantum gravitational corrections[45–47].

In this work, we find a consistent perturbed Hamiltonian formulation. We use Deffayet’s approach [44] to obtain the generalized Hamiltonian of a Galilean theory and along with canonical scalar field Hamiltonian, we perturb both fields to obtain all equations of motion as well as interaction Hamiltonian and we compare with conventional Lagrangian formulation. We find that both lead to identical results and hence, our Hamiltonian approach leads to consistent results in a straightforward and efficient way.

In section 2, we introduce the generic scalar model in the early Universe and briefly discuss gauge fixing and the corresponding gauge invariant equations of motion. In section 3, we take a simple model that highlights the key issues that need to be addressed about the gauge issue in Hamiltonian formulation and also discuss how the same can be addressed. In this simple model, we compare and contrast Lagrangian and Hamiltonian formulation. We calculate third and fourth order perturbed Hamiltonian and replace momenta with the time derivatives of the variables and show that it is consistent with conventional Lagrangian formulation. In section 4, we discuss canonical scalar field in flat-slicing gauge to obtain interaction Hamiltonian in a new and simple way. In Appendix A, we calculate equations of motion of perturbed and unperturbed variables for Canonical scalar field in flat-slicing gauge. We also explicitly obtain the third order interaction Hamiltonian of Canonical scalar field model in phase-space. In order to show that our proposed method works in any gauges, we obtain equations of motion of all variables of Canonical scalar field in uniform density gauge in Appendix B. In section 5 and Appendix C, we extend the analysis to a very specific Galilean field and evaluate all equations of motion of perturbed-unperturbed variables. We show that, at every order unlike higher order generalized Lagrangian, Galilean field model does not provide extra degrees freedom and behave same as any general first order derivative Lagrangian model. We also calculate the third and fourth order perturbed Hamiltonian. In Appendix E, we consider a Galilean model with a canonical scalar field part and express the full Hamiltonian as well as zeroth and second order perturbed Hamiltonian and express zeroth and first order perturbed equations.

In this work, we consider  $(-, +, +, +)$  metric signature. We also denote  $'$  as derivative with respect to conformal time.

## 2 Basic models and Gauge choices

Action for a generic scalar field ( $\varphi$ ) minimally coupled to gravity is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + \mathcal{L}_m(\varphi, \partial\varphi, \partial\partial\varphi) \right], \quad (2.1)$$

where  $R$  is the Ricci scalar and the matter Lagrangian,  $\mathcal{L}_m$  is of the form.

$$\mathcal{L}_m = P(X, \varphi) + G(X, \varphi) \square \varphi, \quad X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi, \quad \square \equiv -\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu). \quad (2.2)$$

It is important to note that the Lagrangian contains second order derivatives, however, the equation of motion will be of second order and these are referred as Galilean[40–42]. Varying the action (2.1) with respect to metric gives Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R = \kappa T_{\mu\nu}, \quad (2.3)$$

where the stress tensor  $T_{\mu\nu}$  is

$$T_{\mu\nu} = g_{\mu\nu} \left( P + G_X g^{\alpha\beta} \partial_\alpha X \partial_\beta \varphi + G_\varphi g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - (P_X + 2G_\varphi + G_X \square \varphi) \partial_\mu \varphi \partial_\nu \varphi - 2G_X \partial_\mu X \partial_\nu \varphi. \quad (2.4)$$

Varying the action (2.1) with respect to the scalar field ' $\varphi$ ' leads to the following equation of motion

$$(2G_\varphi - 2XG_{X\varphi} + P_X) \square \varphi - (P_{XX} + 2G_{X\varphi}) \partial_\mu \varphi \partial^\mu \varphi - 2X (G_\varphi + P_{X\varphi}) + P_\varphi - G_X \left( \varphi_{,\mu\nu} \varphi^{\mu\nu} - (\square \varphi)^2 + R_{\mu\nu} \partial^\mu \varphi \partial^\nu \varphi \right) - G_{XX} (\partial_\mu X \partial^\mu X + \partial_\mu \varphi \partial^\mu X \square \varphi) = 0, \quad (2.5)$$

which can also be obtained by using the conservation of Energy-Momentum tensor,  $\nabla_\mu T^{\mu\nu} = 0$ . Setting  $G(X, \varphi) = 0$  corresponds to non-canonical scalar field. Further, fixing  $P = -X - V(\varphi)$ , where  $V(\varphi)$  is the potential, corresponds to canonical scalar field.

The four-dimensional line element in the ADM form is given by,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -(N^2 - N_i N^i) d\eta^2 + 2N_i dx^i d\eta + \gamma_{ij} dx^i dx^j, \quad (2.6)$$

where  $N(x^\mu)$  and  $N_i(x^\mu)$  are Lapse function and Shift vector respectively,  $\gamma_{ij}$  is the 3-D space metric. Action (2.1) for the line element (2.6) takes the form,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} \left( {}^{(3)}R + K_{ij} K^{ij} - K^2 \right) + \mathcal{L}_m \right\} \quad (2.7)$$

where  $K_{ij}$  is extrinsic curvature tensor and is defined by

$$K_{ij} \equiv \frac{1}{2N} \left[ \partial_0 \gamma_{ij} - N_{i|j} - N_{j|i} \right] \\ K \equiv \gamma^{ij} K_{ij}$$

Perturbatively expanding the metric only in terms of scalar perturbations and the scalar field about the flat FRW spacetime in conformal coordinate, we get,

$$g_{00} = -a(\eta)^2 (1 + 2\epsilon\phi_1 + \epsilon^2\phi_2 + \dots) \quad (2.8)$$

$$g_{0i} \equiv N_i = a(\eta)^2 (\epsilon\partial_i B_1 + \frac{1}{2}\epsilon^2\partial_i B_2 + \dots) \quad (2.9)$$

$$g_{ij} = a(\eta)^2 ((1 - 2\epsilon\psi_1 - \epsilon^2\psi_2 - \dots)\delta^{ij} + 2\epsilon E_{1ij} + \epsilon^2 E_{2ij} + \dots) \quad (2.10)$$

$$\varphi = \varphi_0(\eta) + \epsilon\varphi_1 + \frac{1}{2}\epsilon^2\varphi_2 + \dots \quad (2.11)$$

where  $\epsilon$  denotes the order of the perturbation. To determine the dynamics at every order, we need five scalar functions ( $\phi, B, \psi, E$  and  $\varphi$ ) at each order. Since there are two free gauge

choices, one can fix two of the five scalar functions. In this work, we derive all equations by choosing a specific gauge — flat-slicing gauge, i.e.,  $\psi = 0, E = 0$  — at all orders:

$$g_{00} = -a(\eta)^2(1 + 2\epsilon\phi_1 + \epsilon^2\phi_2 + \dots) \quad (2.12)$$

$$g_{0i} \equiv N_i = a(\eta)^2(\epsilon\partial_i B_1 + \frac{1}{2}\epsilon^2\partial_i B_2 + \dots) \quad (2.13)$$

$$g_{ij} = a(\eta)^2\delta_{ij} \quad (2.14)$$

$$\varphi = \varphi_0(\eta) + \epsilon\varphi_1 + \frac{1}{2}\epsilon^2\varphi_2 + \dots \quad (2.15)$$

It can be shown that, perturbed equations in flat-slicing gauge coincide with gauge-invariant equations of motion (in generic gauge,  $\varphi_1$  coincides with  $\varphi_1 + \frac{\varphi_0'}{H}\psi_1 \equiv \frac{\varphi_0'}{H}\mathcal{R}$  which is a gauge-invariant quantity,  $\mathcal{R}$  is called curvature perturbation). Similarly, one can choose another suitable gauge with no coordinate artifacts to obtain gauge-invariant equations of motion[52]. Such gauges are Newtonian-conformal gauge ( $B = 0, E = 0$ ), constant density gauge ( $E = 0, \delta\varphi = 0$ ), etc.

One immediate question that needs to be addressed in the Hamiltonian formulation is the following: for a given gauge choice, if a particular set of variables are set to zero, whether the corresponding conjugate momenta also vanish? In other words, in the flat-slicing gauge  $\delta g_{ij} = 0$ , does this mean the corresponding canonical conjugate momentum  $\delta\pi^{ij}$  vanish? In order to go about understanding this, in the next section, we take a simple model of two variables ( $x$  and  $y$ ) where one of the variables is perturbed, while the other variable is not perturbed and study the Hamiltonian formulation of this model.

### 3 Simple model: Warm up

As discussed above, in cosmological perturbation theory, by fixing a ‘gauge’, we assume some field variables to be unperturbed where some variables are perturbed. To go about understanding the procedure in the Hamiltonian formulation, we consider a simple classical model that consists of both perturbed and unperturbed variables. We also show that the Hamilton’s equations of unperturbed as well as perturbed variables are identical to Euler-Lagrange equations of motion. The Lagrangian of the simple model is

$$\mathcal{L} = \frac{1}{2y} ((\partial_t x)^2 + (\partial_t y)^2) - \frac{1}{4}(x^4 + y^4). \quad (3.1)$$

The corresponding momenta are

$$\pi_x = \frac{\partial_t x}{y}, \quad \pi_y = \frac{\partial_t y}{y}, \quad (3.2)$$

and the corresponding Hamiltonian (3.1) is given by

$$\mathcal{H} = \frac{1}{2}y(\pi_x^2 + \pi_y^2) + \frac{1}{4}(x^4 + y^4). \quad (3.3)$$

#### 3.1 Perturbed Lagrangian

As mentioned earlier, we consider  $x$  to be unperturbed and  $y$  to be perturbed, and separate into background and perturbed parts, i.e.,

$$x = x_0, \quad y = y_0 + \epsilon y_1. \quad (3.4)$$

where  $\epsilon$  is the order of perturbation. In this work, we mainly focus on first order perturbation, however, the analysis can be extended to any higher order perturbations. Using (3.4), we separate the Lagrangian (3.1) into a background part and perturbed parts, and write it as

$$\mathcal{L} = \mathcal{L}_0 + \epsilon \mathcal{L}_1 + \epsilon^2 \mathcal{L}_2 + \epsilon^3 \mathcal{L}_3 + \epsilon^4 \mathcal{L}_4 + \dots \quad (3.5)$$

where

$$\mathcal{L}_0 = \frac{1}{2} (\partial_t x_0)^2 y_0^{-1} + \frac{1}{2} (\partial_t y_0)^2 y_0^{-1} - \frac{1}{4} (x_0^4 + y_0^4) \quad (3.6)$$

$$\mathcal{L}_1 = \partial_t x_0 \partial_t y_1 y_0^{-1} - \frac{1}{2} (\partial_t x_0)^2 y_0^{(-2)} y_1 - \frac{1}{2} (\partial_t y_0)^2 y_0^{(-2)} y_1 - y_0^3 y_1 \quad (3.7)$$

$$\mathcal{L}_2 = \frac{1}{2} (\partial_t y_1)^2 y_0^{-1} - \partial_t y_0 \partial_t y_1 y_0^{(-2)} y_1 + \frac{1}{2} (\partial_t x_0)^2 y_0^{(-3)} y_1^2 + \frac{1}{2} (\partial_t y_0)^2 y_0^{(-3)} y_1^2 - \frac{3}{2} y_0^2 y_1^2 \quad (3.8)$$

$$\mathcal{L}_3 = -\frac{1}{2} (\partial_t y_1)^2 y_0^{(-2)} y_1 + \partial_t y_0 \partial_t y_1 y_0^{(-3)} y_1^2 - \frac{1}{2} (\partial_t x_0)^2 y_0^{(-4)} y_1^3 - \frac{1}{2} (\partial_t y_0)^2 y_0^{(-4)} y_1^3 - y_1^3 y_0 \quad (3.9)$$

$$\mathcal{L}_4 = \frac{1}{2} (\partial_t y_1)^2 y_0^{(-3)} y_1^2 - \partial_t y_0 \partial_t y_1 y_0^{(-4)} y_1^3 + \frac{1}{2} (\partial_t x_0)^2 y_0^{(-5)} y_1^4 + \frac{1}{2} (\partial_t y_0)^2 y_0^{(-5)} y_1^4 - \frac{1}{4} y_1^4. \quad (3.10)$$

Euler-Lagrange equations of motion of  $x$  and  $y$  for the Lagrangian (3.1) are

$$\partial_t \left( \frac{\partial_t x}{y} \right) + x^3 = 0 \quad (3.11)$$

$$\partial_t \left( \frac{\partial_t y}{y} \right) = -\frac{1}{2y^2} ((\partial_t x)^2 + (\partial_t y)^2) - y^3 \quad (3.12)$$

We perform order-by-order perturbation of the above equation using (3.4) which can also be obtained by varying the perturbed Lagrangian. Zeroth order equations of  $x$ , i.e.,  $x_0$  and  $y$ , i.e.,  $x_0$  are given by

$$\partial_t \left( \frac{\partial_t x_0}{y_0} \right) + x_0^3 = 0 \quad (3.13)$$

$$\partial_t \left( \frac{\partial_t y_0}{y_0} \right) = -\frac{1}{2y_0^2} ((\partial_t x_0)^2 + (\partial_t y_0)^2) - y_0^3. \quad (3.14)$$

One can either perturb the equation (3.12) or vary the second order Lagrangian (3.8) with respect to  $y_1$  to obtain first order perturbed equation of motion of  $y$  and is given by

$$\partial_t \left( \frac{\partial_t y_1}{y_0} - \frac{\partial_t y_0}{y_0^2} y_1 \right) = \frac{1}{y_0^3} ((\partial_t x_0)^2 + (\partial_t y_0)^2) y_1 - \frac{\partial_t y_0}{y_0^2} \partial_t y_1 - 3y_0^2 y_1. \quad (3.15)$$

In the next subsection, we explicitly write down the perturbed equations using Hamiltonian (3.3).

### 3.2 Perturbed Hamiltonian

It is important to note that even though  $x$  is not perturbed,  $\pi_x$  contains both perturbed and unperturbed parts. Using (3.4), the following relations can easily be established<sup>1</sup>

$$\pi_x = \pi_{x0} + \epsilon \pi_{x1} \quad (3.16)$$

$$\pi_y = \pi_{y0} + \epsilon \pi_{y1} \quad (3.17)$$

where

$$\pi_{x0} = \frac{\partial_t x_0}{y_0}, \quad \pi_{x1} = -\frac{y_1}{y_0} \pi_{x0} \quad (3.18)$$

$$\pi_{y0} = \frac{\partial_t y_0}{y_0}, \quad \pi_{y1} = \frac{\partial_t y_1}{y_0} - \frac{y_1}{y_0} \pi_{y0} \quad (3.19)$$

Using (3.4), (3.16) and (3.17), Hamiltonian of the system (3.3) can be written as

$$\mathcal{H} = \mathcal{H}_0 + \epsilon \mathcal{H}_1 + \epsilon^2 \mathcal{H}_2 + \epsilon^3 \mathcal{H}_3 + \epsilon^4 \mathcal{H}_4 + \dots \quad (3.20)$$

where

$$\mathcal{H}_0 = \frac{1}{2} y_0 (\pi_{x0}^2 + \pi_{y0}^2) + \frac{1}{4} (x_0^4 + y_0^4) \quad (3.21)$$

$$\mathcal{H}_1 = \frac{1}{2} \pi_{x0}^2 y_1 + \pi_{x0} \pi_{x1} y_0 + \frac{1}{2} \pi_{y0}^2 y_1 + \pi_{y0} \pi_{y1} y_0 + y_0^3 y_1 \quad (3.22)$$

$$\mathcal{H}_2 = y_1 (\pi_{x0} \pi_{x1} + \pi_{y0} \pi_{y1}) + \frac{1}{2} y_0 (\pi_{x1}^2 + \pi_{y1}^2) + \frac{3}{2} y_0^2 y_1^2 \quad (3.23)$$

$$\mathcal{H}_3 = \frac{1}{2} y_1 (\pi_{x1}^2 + \pi_{y1}^2) + y_1^3 y_0 \quad (3.24)$$

$$\mathcal{H}_4 = \frac{1}{4} y_1^4 \quad (3.25)$$

Using (3.21), we obtain zeroth order Hamilton's equations

$$\partial_t x_0 = y_0 \pi_{x0}, \quad \partial_t \pi_{x0} = -x_0^3 \quad (3.26)$$

$$\partial_t y_0 = y_0 \pi_{y0}, \quad \partial_t \pi_{y0} = -\frac{1}{2} ((\pi_{x0}^2 + \pi_{y0}^2) - y_0^3) \quad (3.27)$$

Using (3.23), first order Hamilton's equations are

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<sup>1</sup>It is apparent from equations (3.2) that only first order perturbation of  $y$  can produce any higher order perturbed momenta of  $x$  and  $y$  and it is given by

$$\begin{aligned} \pi_x &= \pi_{x0} + \epsilon \pi_{x1} + \epsilon^2 \pi_{x2} \dots \\ \pi_y &= \pi_{y0} + \epsilon \pi_{y1} + \epsilon^2 \pi_{y2} \dots \end{aligned}$$

where

$$\begin{aligned} \pi_{x0} &= \frac{d_t x_0}{y_0}, \quad \pi_{x1} = -\frac{y_1}{y_0} \pi_{x0}, \quad \pi_{x2} = -\frac{y_1}{y_0} \pi_{x1}, \quad \pi_{x3} = -\frac{y_1}{y_0} \pi_{x2} \quad \dots \\ \pi_{y0} &= \frac{d_t y_0}{y_0}, \quad \pi_{y1} = \frac{d_t y_1}{y_0} - \frac{y_1}{y_0} \pi_{y0}, \quad \pi_{y2} = -\frac{y_1}{y_0} \pi_{y1}, \quad \pi_{y3} = -\frac{y_1}{y_0} \pi_{y2} \quad \dots \end{aligned}$$

Since we are not interested in higher order perturbation theory, we neglect higher order momenta and consider only first order momenta in calculating correlation functions.



$$\frac{\partial \mathcal{H}_2}{\partial \pi_{x1}} = 0 \quad \Rightarrow \quad \pi_{x0}y_1 + \pi_{x1}y_0 = 0 \quad (3.28)$$

$$\partial_t y_1 = \frac{\partial \mathcal{H}_2}{\partial \pi_{y1}} = \pi_{y0}y_1 + \pi_{y1}y_0 \quad (3.29)$$

$$\partial_t \pi_{y1} = -(\pi_{x0}\pi_{x1} + \pi_{y0}\pi_{y1}) - 3y_0^2 y_1 \quad (3.30)$$

Equation (3.28) gives explicit expression for  $\pi_{x1}$  and leads to identical expression as in (3.16). It can easily be verified that, zeroth order equations (3.26) and (3.27) are identical to equations (3.13) and (3.14), respectively, where (3.28), (3.29) and (3.30) lead to the equivalent equation of motion of  $y_1$  (3.15).

To compare the the above expression with that from the Lagrangian formulation, we rewrite the above expressions using (3.16) and (3.16). We get

$$\mathcal{H}_0 = \frac{1}{2} (\partial_t x_0)^2 y_0^{-1} + \frac{1}{2} (\partial_t y_0)^2 y_0^{-1} + \frac{1}{4} x_0^4 + \frac{1}{4} y_0^4 \quad (3.31)$$

$$\mathcal{H}_1 = -\frac{1}{2} (\partial_t x_0)^2 y_0^{(-2)} y_1 - \frac{1}{2} (\partial_t y_0)^2 y_0^{(-2)} y_1 + \partial_t y_0 \partial_t y_1 y_0^{-1} + y_0^3 y_1 \quad (3.32)$$

$$\mathcal{H}_2 = -\frac{1}{2} (\partial_t x_0)^2 y_0^{(-3)} y_1^2 - \frac{1}{2} (\partial_t y_0)^2 y_0^{(-3)} y_1^2 + \frac{1}{2} (\partial_t y_1)^2 y_0^{-1} + \frac{3}{2} y_0^2 y_1^2 \quad (3.33)$$

$$\mathcal{H}_3 = \frac{1}{2} (\partial_t x_0)^2 y_0^{(-4)} y_1^3 + \frac{1}{2} (\partial_t y_0)^2 y_0^{(-4)} y_1^3 - \partial_t y_0 \partial_t y_1 y_0^{(-3)} y_1^2 + \frac{1}{2} (\partial_t y_1)^2 y_0^{(-2)} y_1 + y_1^3 y_0 \quad (3.34)$$

$$\mathcal{H}_4 = \frac{1}{4} y_1^4 \quad (3.35)$$

It is important to note that, only the third order perturbed Hamiltonian is negative of the third order Lagrangian, i.e.,

$$\mathcal{H}_3 = -\mathcal{L}_3. \quad (3.36)$$

Explicit forms of Interaction Hamiltonians can be obtained using perturbed parts of the Lagrangian[43] and using (3.8), (3.9) and (3.10), it can be verified that, both approaches lead to the identical results. In this approach, to obtain  $n^{th}$  order interaction Hamiltonian, the Lagrangian is expanded up to  $n^{th}$  order perturbation and by varying the Lagrangian, the momentum corresponding to the perturbed quantity is obtained as a non-linear combination of time derivative of the field. Using perturbation techniques, this relation is inverted and the time derivative of the field is written in terms of non-linear combination of corresponding momentum. Perturbed Hamiltonian corresponding to the perturbed parts of the Lagrangian is obtained by using the conventional definition of Hamiltonian as  $\mathcal{H} = \pi \dot{\phi} - \mathcal{L}$  and  $\dot{\phi}$  is replaced by the above relation. Once the perturbed Hamiltonian is obtained in terms of field variable and conjugate momentum, to calculate correlation functions, the momentum in the Hamiltonian is replaced in terms of time derivative of the field and in this time, the linear relation of momentum and time derivative of the field is used. The above procedure is rather cumbersome and involves series of approximations. Since we start with the general Hamiltonian, our approach is very straightforward and efficient.

As mentioned earlier, while we focus on first order perturbations, our approach can easily be extended to any higher order perturbation, e.g., to obtain second order equations of motion of field variables, we have to consider up to second order field perturbation and its corresponding momentum up to second order and calculate the fourth order perturbed Hamiltonian. Since, we already have obtained zeroth and first order equations, second order perturbed field equations are obtained by varying fourth order perturbed Hamiltonian with respect to second order perturbed variables and their corresponding momenta.

## 4 Canonical scalar field

In order to show the advantages of the Hamiltonian formulation, we first focus on canonical scalar field. The action (2.7) for canonical scalar field in the ADM formulation is

$$\begin{aligned} \mathcal{S}_C = \int d^4x & \left[ N \frac{\gamma^{\frac{1}{2}}}{2\kappa} \left( {}^{(3)}R + K_{ij}K^{ij} - K^2 \right) + \frac{1}{2} N^{-1} \gamma^{\frac{1}{2}} (\partial_0 \varphi)^2 - N^i \partial_0 \varphi \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}} \right. \\ & \left. - \frac{1}{2} N \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} + \frac{1}{2} N^i N^j \partial_i \varphi \partial_j \varphi N^{-1} \gamma^{\frac{1}{2}} - N V \gamma^{\frac{1}{2}} \right]. \end{aligned} \quad (4.1)$$

To obtain the equations of motion of all variables, one can simply use the Einstein's equation or one can directly vary the action with respect to field variables. 0-0 and 0-i components of the Einstein's equations represent the equations of motion of  $g_{00}$  and  $g_{0i}$ , which is  $N$  and  $N^i$  respectively. Hence, the above two equations are identical to Hamiltonian and Momentum constraints respectively and i-j component of the Einstein's equations and conservation of Energy-Momentum tensor lead to equation of motion of 3-metric and matter field, respectively. In the rest of this section, we will use the definitions (2.12), (2.13), (2.14) and (2.15) in the the above equations to obtain perturbed equations of motion of gravitational field variables at all order. In Appendix A, we derive zeroth and first order perturbed field equations of canonical scalar field and in the following subsection, we apply the procedure discussed in section 3 to canonical scalar field model to obtain consistent equations of motion as well as interaction Hamiltonian using Hamiltonian formulation.

### 4.1 Hamiltonian formulation

Conjugate momenta of all field variables  $\gamma_{ij}$ ,  $\varphi$ ,  $N$  and  $N^i$  are defined as

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{\gamma}_{ij}}, \quad \pi_\varphi \equiv \frac{\delta L}{\delta \dot{\varphi}}, \quad \pi_N \equiv \frac{\delta L}{\delta \dot{N}}, \quad \pi_i \equiv \frac{\delta L}{\delta \dot{N}^i}. \quad (4.2)$$

Using the action (4.1), conjugate momenta are given by

$$\pi^{ij} = \frac{1}{2} \kappa^{-1} \gamma^{\frac{1}{2}} (\gamma^{ij} \gamma^{kl} - \gamma^{ik} \gamma^{jl}) K_{kl} \quad (4.3)$$

$$\pi_\varphi = N^{-1} \gamma^{\frac{1}{2}} \varphi' - N^i \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}} \quad (4.4)$$

$$\Phi^N \equiv \pi_N = 0, \quad \Phi_i^{N^i} \equiv \pi_i = 0. \quad (4.5)$$

From equation (4.5), it is apparent that, for canonical scalar field, Lapse function  $N$  and shift vector  $N^i$  are constraints and behave like Lagrange multipliers and it gives 4-primary constraint relations. Inverse relations of equations (4.3) and (4.4) are

$$\gamma'_{mn} = \gamma_{nk} N^k|_m + \gamma_{mk} N^k|_n - 2 N K_{mn}, \quad K_{ij} \equiv \kappa \gamma^{-\frac{1}{2}} (\gamma_{ij} \gamma_{kl} - 2 \gamma_{ik} \gamma_{jl}) \pi^{kl} \quad (4.6)$$

$$\varphi' = N \pi_\varphi \gamma^{-\frac{1}{2}} + N^i \partial_i \varphi \quad (4.7)$$

Using the above definitions of canonical momenta and (4.1), we get the Hamiltonian density of the system

$$\begin{aligned}
\mathcal{H}_C &= \pi^{ij} \gamma'_{ij} + \pi_\varphi \varphi' - L \\
&= \gamma_{jk} \partial_i N^k \pi^{ij} + N^k \partial_k \gamma_{ij} \pi^{ij} + \gamma_{ik} \partial_j N^k \pi^{ij} - N \gamma_{ij} \gamma_{kl} \kappa \pi^{kl} \pi^{ij} \gamma^{-\frac{1}{2}} + \\
&\quad 2 N \gamma_{ik} \gamma_{jl} \kappa \pi^{kl} \pi^{ij} \gamma^{-\frac{1}{2}} + \frac{1}{2} N \pi_\varphi^2 \gamma^{-\frac{1}{2}} + N^i \pi_\varphi \partial_i \varphi - \frac{1}{2} N^{(3)} R \gamma^{\frac{1}{2}} \kappa^{-1} + \\
&\quad \frac{1}{2} N \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} + N V \gamma^{\frac{1}{2}}.
\end{aligned} \tag{4.8}$$

Since the action (4.1) has diffeomorphism-invariance, Hamiltonian (4.8) vanishes identically, i.e.,

$$\mathcal{H}_C = 0.$$

Evolution of primary constraints vanishes weakly and gives rise to four secondary constraints: one Hamiltonian constraint due to  $\mathcal{H}_N \equiv \frac{d\Phi^N}{dt} = \{\pi_N, \mathcal{H}_C\} \equiv -\frac{\delta \mathcal{H}_C}{\delta N} \approx 0$  and three Momentum constraints due to  $\mathcal{H}_i \equiv \frac{d\Phi_i}{dt} = \{\pi_i, \mathcal{H}_C\} \equiv -\frac{\delta \mathcal{H}_C}{\delta N^i} \approx 0$ .

$$\begin{aligned}
\mathcal{H}_N &\equiv N \kappa \gamma^{-\frac{1}{2}} (2 \gamma_{ik} \gamma_{jl} - \gamma_{ij} \gamma_{kl}) \kappa \pi^{kl} \pi^{ij} + \frac{1}{2} N \pi_\varphi^2 \gamma^{-\frac{1}{2}} \\
&\quad - \frac{1}{2} N^{(3)} R \gamma^{\frac{1}{2}} \kappa^{-1} + \frac{1}{2} N \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} + N V \gamma^{\frac{1}{2}} \approx 0
\end{aligned} \tag{4.9}$$

$$\mathcal{H}_i \equiv -2 \partial_k \gamma_{ij} \pi^{jk} + \pi^{jk} \partial_i \gamma_{jk} + \pi_\varphi \partial_i \varphi \approx 0. \tag{4.10}$$

Hamiltonian density can be written in terms Hamiltonian and Momentum constraint as

$$\mathcal{H}_C = N \mathcal{H}_N + N^i \mathcal{H}_i \approx 0. \tag{4.11}$$

#### 4.1.1 Zeroth order Hamilton's equations

Using  $\gamma_{ij} = a^2 \delta_{ij}$  and all background quantities being independent of spatial coordinates, Hamiltonian density (4.8) becomes

$$\mathcal{H}_0^C = 2 N_0 a \kappa (\delta_{ik} \delta_{jl} - \delta_{ij} \delta_{kl}) \pi_0^{ij} \pi_0^{kl} + \frac{1}{2} N_0 \pi_{\varphi 0}^2 a^{(-3)} + N_0 V a^3 \approx 0. \tag{4.12}$$

At zeroth order, conjugate momentum of  $a$ ,  $\pi_a \equiv \frac{\delta L}{\delta a}$  is directly related with  $\pi_0^{ij}$  by the simple relation  $\pi_0^{ij} = \frac{1}{6a} \pi_a \delta^{ij}$ ,  $\pi_a = 2a \delta_{ij} \pi_0^{ij}$ . The zeroth order Hamiltonian (4.12), in terms of  $\pi_a$  takes the simple form

$$\mathcal{H}_0^C = N_0 \left[ -\frac{1}{12} \kappa \pi_a^2 a^{-1} + \frac{1}{2} \pi_{\varphi 0}^2 a^{(-3)} + V a^3 \right] \equiv N_0^{(0)} \mathcal{H}_N. \tag{4.13}$$

The terms inside the bracket in the right hand side is the Hamiltonian constraint. At zeroth order it is independent of  $N_0$ . So, as we have mentioned earlier,  $N_0$  cannot be determined uniquely and we can choose it arbitrarily. In this work, we use comoving coordinate,  $N_0 = a$ .

Varying the Hamiltonian (4.13) with respect to the momenta, we get

$$a' = -1/6 N_0 \kappa \pi_a a^{-1} \tag{4.14}$$

$$\Rightarrow \pi_a = -6 a a' N_0^{-1} \kappa^{-1} \Rightarrow \pi_0^{ij} = -N_0^{-1} \kappa^{-1} a' \delta^{ij} \tag{4.15}$$

$$\varphi'_0 = N_0 \pi_{\varphi 0} a^{(-3)} \tag{4.16}$$

Hamiltonian constraint in conformal or comoving coordinate in terms of field derivatives is

$$\mathcal{H}_{N0} \equiv -\frac{1}{12} \kappa \pi_a^2 a^{-1} + \frac{1}{2} \pi_{\varphi 0}^2 a^{(-3)} + V a^3 = 0 \quad (4.17)$$

$$\Rightarrow -3 \kappa^{-1} H^2 + \frac{1}{2} \varphi_0'^2 + V a^2 = 0, \quad \text{where} \quad H \equiv \frac{a'}{a} \quad (4.18)$$

Variation of the Hamiltonian with respect to the field variables and relating with the time derivatives of the momenta lead to the dynamical equation of motion of the field variables. Hence, equation of motion of  $a$  in comoving coordinate in terms of field derivatives becomes

$$\begin{aligned} \pi_a' + \frac{\delta \mathcal{H}_0^C}{\delta a} &= 0 \\ \Rightarrow \pi_a' + \frac{1}{12} N_0 \kappa \pi_a^2 a^{(-2)} - \frac{3}{2} N_0 \pi_{\varphi 0}^2 a^{(-4)} + 3 N_0 V a^2 &= 0 \end{aligned} \quad (4.19)$$

$$\Rightarrow 3 \kappa^{-1} H^2 - 6 \frac{a''}{a} \kappa^{-1} - \frac{3}{2} \varphi_0'^2 a + 3 V a^2 = 0. \quad (4.20)$$

Similarly, the equation of motion of  $\varphi_0$  takes the form

$$\pi_{\varphi 0}' + N_0 V_{\varphi} a^3 = 0 \quad (4.21)$$

$$\Rightarrow \varphi_0'' + 2H \varphi_0' + V_{\varphi} a^2 = 0. \quad (4.22)$$

The three equations (4.18), (4.20) and (4.22) are, as expected, identical to the equations (A.1), (A.2) and (A.3) respectively.

#### 4.1.2 First order perturbed Hamilton's equation

As mentioned earlier, we consider flat-slicing gauge, hence there is no perturbation in the 3-metric, i.e.,  $\delta g_{ij} = 0$ . As we pointed out in the simple model, while  $x$  is treated as unperturbed,  $\pi_x$  is non-zero. In the flat-slicing gauge, there is no perturbation in the 3-metric, however, canonical conjugate momentum corresponding to 3-metric will have non-zero perturbed contributions. This becomes transparent if we perturb (4.3), i.e.,

$$\delta \pi^{mn} = \frac{1}{2} \kappa^{-1} \gamma^{\frac{1}{2}} (\gamma^{mn} \gamma^{kl} - \gamma^{mk} \gamma^{nl}) \delta K_{kl}.$$

Hence, perturbed part of  $K_{ij}$ , i.e.,  $\delta K_{ij}$  is not zero and it contributes to the perturbed part of  $\pi^{ij}$ , i.e.,  $\delta \pi^{ij}$ .

We can separate unperturbed and perturbed parts of field variables and their corresponding momenta as

$$N = N_0 + \epsilon N_1, \quad N^i = \epsilon N_1^i, \quad \varphi = \varphi_0 + \epsilon \varphi_1 \quad (4.23)$$

$$\pi^{ij} = \pi_0^{ij} + \epsilon \pi_1^{ij}, \quad \pi_{\varphi} = \pi_{\varphi 0} + \epsilon \pi_{\varphi 1} \quad (4.24)$$

Comparing (2.12) and (2.13) with  $N_1$  and  $N_1^i$ , we obtain

$$N_1 = a \phi_1, \quad N_1^i = \delta^{ij} \partial_{ij} B_1 \quad (4.25)$$

The second order Hamiltonian density can be obtained by substituting (4.23) and (4.24) in (4.8)

$$\begin{aligned}\mathcal{H}_2^C = & \delta_{ij}\partial_k N_1^j \pi_1^{ik} a^2 + \delta_{ij}\partial_k N_1^j \pi_1^{ik} a^2 - N_0 \delta_{ij} \delta_{kl} \kappa \pi_1^{ij} \pi_1^{kl} a - \\ & 2 N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_1^{kl} a + 2 N_0 \delta_{ij} \delta_{kl} \kappa \pi_1^{ik} \pi_1^{jl} a + 4 N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_1^{jl} a + \\ & \frac{1}{2} N_0 \pi_{\varphi 1}^2 a^{(-3)} + N_1 \pi_{\varphi 0} \pi_{\varphi 1} a^{(-3)} + N_1^i \pi_{\varphi 0} \partial_i \varphi_1 + \\ & \frac{1}{2} N_0 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a + N_1 V_{\varphi} a^3 \varphi_1 + \frac{1}{2} N_0 V_{\varphi \varphi} \varphi_1^2 a^3.\end{aligned}\quad (4.26)$$

Since there is no perturbation in the 3-metric  $\gamma_{ij}$ , variation with respect to  $\pi_1^{ij}$  gives rise to a separate constraint equation from which we can extract the derived value of  $\pi_1^{ij}$ .

$$\frac{\delta \mathcal{H}_2^C}{\delta \pi_1^{ij}} = 0 \quad (4.27)$$

$$\begin{aligned}\Rightarrow & \delta_{nj} \partial_m N_1^j a^2 + \delta_{mj} \partial_n N_1^j a^2 - 2 N_0 \delta_{mn} \delta_{kl} \kappa \pi_1^{kl} a + 4 N_0 \delta_{mk} \delta_{nl} \kappa \pi_1^{kl} a - \\ & 2 N_1 \delta_{mn} \delta_{kl} \kappa \pi_0^{kl} a + 4 N_1 \delta_{mk} \delta_{nl} \kappa \pi_0^{kl} a = 0.\end{aligned}\quad (4.28)$$

Multiplying above expression with  $(\delta^{mn} \delta^{ij} - \delta^{mi} \delta^{nj})$  gives

$$\begin{aligned}\pi_1^{ij} = & \frac{1}{2} N_0^{-1} \kappa^{-1} a \delta^{ij} \partial_k N_1^k - \frac{1}{4} N_0^{-1} \kappa^{-1} a \delta^{ki} \partial_k N_1^j - \frac{1}{4} N_0^{-1} \kappa^{-1} a \delta^{kj} \partial_k N_1^i \\ & - N_0^{-1} N_1 \pi_0^{ij}.\end{aligned}\quad (4.29)$$

The relation of time derivative of perturbed matter field  $\varphi_1$  and conjugate momentum of  $\varphi_1$ ,  $(\pi_{\varphi 1})$  are

$$\begin{aligned}\varphi_1' = & \frac{\delta \mathcal{H}_2^C}{\delta \pi_{\varphi 1}} \\ \Rightarrow \varphi_1' = & N_0 \pi_{\varphi 1} a^{(-3)} + N_1 \pi_{\varphi 0} a^{(-3)}.\end{aligned}\quad (4.30)$$

In the conformal coordinate, the perturbed Hamiltonian constraint is obtained by varying the second order perturbed Hamiltonian density (4.26) with respect to perturbed Lapse function  $N_1$ . Using above relations for the momenta with the time derivatives of the field variables and (4.25), the perturbed Hamiltonian constraint becomes

$$\frac{\delta \mathcal{H}_2^C}{\delta N_1} = 0 \quad (4.31)$$

$$\begin{aligned}\Rightarrow & 2 \delta^{ij} H \partial_{ij} B_1 \kappa^{-1} + 6 \phi_1 \kappa^{-1} H^2 + \varphi_0' \varphi_1' - \\ & \phi_1 \varphi_0'^2 + V_{\varphi} a^2 \varphi_1 = 0.\end{aligned}\quad (4.32)$$

Similarly, Momentum constraint is given by

$$M_i \equiv \frac{\delta \mathcal{H}_2^C}{\delta N_1^i} = 0 \quad (4.33)$$

$$\Rightarrow \varphi_0' \partial_i \varphi_1 - 2 H \partial_i \phi_1 \kappa^{-1} = 0. \quad (4.34)$$

and the equation of motion of the perturbed scalar field  $\varphi_1$  becomes

$$\pi_{\varphi_1}' + \frac{\delta \mathcal{H}_2^C}{\delta \varphi_1} = 0 \quad (4.35)$$

$$\begin{aligned} \Rightarrow \varphi_1'' + 2H \varphi_1' - \phi_1' \varphi_0' + 2V_\varphi \phi_1 a^2 - \delta^{ij} \varphi_0' \partial_{ij} B_1 \\ - \delta^{ij} \partial_{ij} \varphi_1 + V_{\varphi\varphi} a^2 \varphi_1 = 0. \end{aligned} \quad (4.36)$$

Equation (4.32), (4.34) and (4.36) are identical to the equations (A.4), (A.5) and (A.7), respectively.

This is a very important result. Unlike Lagrangian formalism, choosing a gauge in Hamiltonian formalism is not trivial. But using the above simple mechanism, it is possible to construct the perturbed Hamiltonian and its equations of motion and can now be treated in the same manner as Lagrangian formalism. It can, even, be extended to any order of perturbation, e.g., to obtain second order equations of motion of field variables, we have to extend the Hamiltonian at fourth order perturbation in terms of second order field variables and the second order momenta and vary the Hamiltonian with respect to second order variables and its conjugate momenta. At second order, in flat-slicing gauge, we will again obtain a constraint equation  $\frac{\partial \mathcal{H}_4}{\partial \pi_2^{ij}} = 0$  that will give the expression of  $\pi_2^{ij}$ .

Our proposed mechanism works for any other arbitrary gauge also. In Appendix B, we obtain all consistent perturbed and unperturbed Hamilton's equations for Canonical scalar field in uniform density gauge. This mechanism can be applied to any generalized first order derivative theory like non-canonical scalar field.

#### 4.1.3 Single variable-Momentum Hamiltonian

Since the second order Hamiltonian holds the dynamics of first order perturbed variables, we can obtain a single variable-single momentum effective Hamiltonian of the background and constraint equations, and replacing background momenta in terms of time derivatives of the fields. Substituting  $\pi_1^{ij}$  using (4.29) and  $\pi_0^{ij}$  and  $\pi_{\varphi_0}$  in the second order Hamiltonian and using (4.25), we get

$$\begin{aligned} \mathcal{H}_2^C = & \frac{1}{2} \delta^{ij} \delta^{kl} \partial_{ij} B_1 \partial_{kl} B_1 \kappa^{-1} a^2 - \frac{1}{2} \delta^{ij} \delta^{kl} \partial_{ik} B_1 \partial_{jl} B_1 \kappa^{-1} a^2 + 2 \delta^{ij} H \partial_{ij} B_1 \phi_1 \kappa^{-1} a^2 \\ & + 3 \kappa^{-1} H^2 \phi_1^2 a^2 + \frac{1}{2} \pi_{\varphi_1}^2 a^{(-2)} + \pi_{\varphi_1} \varphi_0' \phi_1 + \delta^{ij} \varphi_0' \partial_i B_1 \partial_j \varphi_1 a^2 \\ & + \frac{1}{2} \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a^2 + V_\varphi \phi_1 a^4 \varphi_1 + \frac{1}{2} V_{\varphi\varphi} \varphi_1^2 a^4 \end{aligned} \quad (4.37)$$

First and second terms in the right hand side lead to boundary term. Moreover, performing integration by-parts to the seventh term and substituting

$$\phi_1 = \frac{\kappa}{2H} \varphi_0' \varphi_1$$

in the Hamiltonian, we get

$$\begin{aligned} \mathcal{H}_2^C = & \frac{3}{4} \kappa \varphi_0'^2 \varphi_1^2 a^2 + \frac{1}{2} \pi_{\varphi_1}^2 a^{(-2)} + \frac{\kappa}{2H} \pi_{\varphi_1} \varphi_0'^2 \varphi_1 + \frac{1}{2} \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a^2 \\ & + \frac{\kappa}{2H} V_\varphi \varphi_0' \varphi_1^2 a^4 + \frac{1}{2} V_{\varphi\varphi} \varphi_1^2 a^4 \end{aligned} \quad (4.38)$$

This is the single variable-single momentum Hamiltonian density ( $\varphi_1, \pi_{\varphi_1}$ ). Further, this can be expressed in terms of Mukhanov-Sasaki variable-Momentum<sup>2</sup> form. In flat-slicing gauge, Mukhanov-Sasaki variable  $u_1 = a\varphi_1$ , hence,

$$\pi_{\varphi_1} = a\pi_{u_1}.$$

Hence the Hamiltonian in terms of Mukhanov-Sasaki variable and its conjugate momenta takes the form<sup>3</sup>

$$\begin{aligned}\mathcal{H}_2^u &= \mathcal{H}_2^C(u) + \frac{a'}{a} \pi_{u_1} u_1 \\ &= \frac{3}{4} \kappa u_1^2 \varphi_0'^2 + \frac{1}{2} \pi_{u_1}^2 + \frac{1}{2} \pi_{u_1} u_1 \kappa \varphi_0'^2 H^{-1} + \frac{1}{2} \delta^{ij} \partial_i u_1 \partial_j u_1 \\ &\quad + \frac{1}{2} \kappa V_{\varphi} \varphi_0' u_1^2 H^{-1} a^2 + \frac{1}{2} V_{\varphi\varphi} u_1^2 a^2 + H \pi_{u_1} u_1.\end{aligned}\tag{4.39}$$

One can verify that the equation of motion of  $u_1$  becomes

$$u_1'' - \nabla^2 u_1 - \frac{z''}{z} u_1 = 0, \quad \text{where } z \equiv \frac{a\varphi_0'}{H}.$$

#### 4.1.4 Interaction Hamiltonian for calculating higher order correlations

Expanding Hamiltonian (4.8) to third order, we obtain third order Interaction Hamiltonian whose explicit form in phase space is given in Appendix A.3. Replacing the momenta in terms of terms of time derivatives of the fields, interaction Hamiltonian for canonical scalar field becomes

$$\begin{aligned}\mathcal{H}_3^C &= -\frac{1}{2} \delta^{ij} \delta^{kl} \partial_{ij} B_1 \partial_{kl} B_1 \phi_1 \kappa^{-1} a^2 - 2 \delta^{ij} H \partial_{ij} B_1 \kappa^{-1} \phi_1^2 a^2 - 3 \kappa^{-1} H^2 \phi_1^3 a^2 + \\ &\quad \frac{1}{2} \delta^{ij} \delta^{kl} \partial_{ik} B_1 \partial_{jl} B_1 \phi_1 \kappa^{-1} a^2 + \frac{1}{2} \phi_1 \varphi_1'^2 a^2 - \varphi_0' \varphi_1' \phi_1^2 a^2 + \frac{1}{2} \varphi_0'^2 \phi_1^3 a^2 + \\ &\quad \delta^{ij} \varphi_1' \partial_i B_1 \partial_j \varphi_1 a^2 - \delta^{ij} \varphi_0' \partial_i B_1 \partial_j \varphi_1 \phi_1 a^2 + \frac{1}{2} \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 \phi_1 a^2 + \frac{1}{2} V_{\varphi\varphi} \phi_1 \varphi_1^2 a^4 + \\ &\quad \frac{1}{6} V_{\varphi\varphi\varphi} \varphi_1^3 a^4.\end{aligned}\tag{4.40}$$

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<sup>2</sup> Mukhanov-Sasaki variable is also a gauge invariant quantity related to curvature perturbation. At first order, it is given by

$$u_1 = \frac{a\varphi_0'}{H} \mathcal{R}_1.$$

<sup>3</sup> Lagrangian can be written as

$$\mathcal{L} = \pi_{\varphi} \varphi' - \mathcal{H},$$

Changing of variables  $\varphi \rightarrow \frac{u}{a}, \pi_{\varphi} \rightarrow a\pi_u$  leaves the Lagrangian unchanged. Hence,

$$\begin{aligned}\mathcal{L} &= a\pi_u \left( \frac{u'}{a} - \frac{a'}{a^2} u \right) - \mathcal{H} \\ &= \pi_u u' - \frac{a'}{a} u \pi_u - \mathcal{H}\end{aligned}$$

Hence the new Hamiltonian in terms of  $u$  and  $\pi_u$  takes the form

$$\mathcal{H}^u = \mathcal{H}(u) + \frac{a'}{a} u \pi_u$$

It can be verified that  ${}^{(3)}\mathcal{H}_C = -{}^{(3)}\mathcal{L}_C$ . Similarly, fourth order Interaction Hamiltonian for canonical scalar field takes the form

$$\mathcal{H}_4^C = \frac{1}{6} \phi_1 V_{\varphi\varphi\varphi} \varphi_1^3 a^4 + \frac{1}{24} V_{\varphi\varphi\varphi\varphi} \varphi_1^4 a^4 \quad (4.41)$$

which is independent of kinetic part (time derivatives of fields) of the field. This can be verified by looking at the Hamiltonian (4.8). Terms containing momenta only contribute up to third order Hamiltonian since  $\gamma_{ij}$  is unperturbed. Hence, fourth or higher order perturbed Hamiltonian is independent of the kinetic part of the fields. Furthermore, the higher order interaction Hamiltonian can be expressed as single variable form by using constrained equations (4.32) and (4.34).

## 5 Galilean single scalar field model

In the action (2.2),  $G(X, \varphi) \neq 0$  leads to Galilean field where action contains second derivative terms of the field variables. In this present work, to simplify our calculations, we take a specific and simple form of the Galilean model with  $P(X, \varphi) = -V(\varphi)$  and  $G(X, \varphi) = -2X$ , i.e.,

$$\mathcal{S}_G = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R - g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \square \varphi - V(\varphi) \right] \equiv \int \mathcal{L}_G d^4x. \quad (5.1)$$

Zeroth and first order perturbed Euler-Lagrange equations of motion are provided in Appendix C.

### 5.1 Hamiltonian formulation of the Galilean scalar field

Hamiltonian formulation of Higher derivative fields is not unique and there exist several ways [38, 39, 44, 49–51] to rewrite the Hamiltonian as there are infinite ways to absorb the higher derivative terms. For our case, one easy way is to let  $S \equiv \square \varphi$  and re-write the action (5.1) as

$$\mathcal{S}_G = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi S - V(\varphi) \right] + \int d^4x \lambda (S - \square \varphi). \quad (5.2)$$

where  $\lambda$  is the Langrange multiplier whose variation leads to  $S = \square \varphi$ . Since  $\square \varphi$  appears linearly in the action, we can rewrite the action in terms of first order derivatives of the fields by performing integration by-parts as

$$\begin{aligned} \mathcal{S}_G &= \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi S - V(\varphi) \right] + \int d^4x \lambda (S - \square \varphi) \\ &= \dots + \int d^4x \left[ \lambda S - \lambda g^{\mu\nu} (\partial_{\mu\nu} \varphi - \Gamma_{\mu\nu}^\alpha \partial_\alpha \varphi) \right] \\ &= \dots + \int d^4x \left[ \lambda S + \lambda g^{\mu\nu} \Gamma_{\mu\nu}^\alpha \partial_\alpha \varphi + g^{\mu\nu} \partial_\mu \varphi \partial_\nu \lambda + \lambda \partial_\nu g^{\mu\nu} \partial_\mu \varphi \right] + \text{Boundary term} \\ &= \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi S - V(\varphi) \right] + \int d^4x \left[ \lambda S + \lambda g^{\mu\nu} \Gamma_{\mu\nu}^\alpha \partial_\alpha \varphi + \right. \\ &\quad \left. g^{\mu\nu} \partial_\mu \varphi \partial_\nu \lambda + \lambda \partial_\nu g^{\mu\nu} \partial_\mu \varphi \right]. \end{aligned} \quad (5.3)$$



Although the action, now, contains extra variables  $\lambda$  and  $S$ , action (5.1) and (5.3) lead to equivalent equations of motion. Since, the action (5.3) contains no double derivative terms, we can construct the Hamiltonian of the system.

Using the line element (2.6), the action (5.3) can be decomposed and rewritten as

$$\begin{aligned}
\mathcal{S}_G \equiv \int \mathcal{L}_G d^4x = \int d^4x & (S\lambda - NV\gamma^{\frac{1}{2}} + \gamma^{ij}\partial_i\lambda\partial_j\varphi + \lambda\partial_i\gamma^{ij}\partial_j\varphi - \lambda'\varphi'N^{(-2)} + \frac{1}{2}N^{(3)}R\gamma^{\frac{1}{2}}\kappa^{-1} + \\
& N^i\lambda'\partial_i\varphi N^{(-2)} + N^i\varphi'\partial_i\lambda N^{(-2)} + SN^{-1}\gamma^{\frac{1}{2}}\varphi'^2 + \lambda N'\varphi'N^{(-3)} - N^iN^j\partial_i\lambda\partial_j\varphi N^{(-2)} - \\
& N^i\lambda N'\partial_i\varphi N^{(-3)} - N^i\lambda\varphi'\partial_iN N^{(-3)} + \gamma^{ij}\gamma^{kl}\lambda\partial_i\gamma_{jk}\partial_l\varphi - \frac{1}{2}\gamma^{ij}\gamma^{kl}\lambda\partial_i\gamma_{kl}\partial_j\varphi + \\
& \frac{1}{2}\gamma^{ij}\lambda\gamma_{ij}'\varphi'N^{(-2)} - \gamma^{ij}\lambda\partial_iN\partial_j\varphi N^{-1} - NS\gamma^{ij}\partial_i\varphi\partial_j\varphi\gamma^{\frac{1}{2}} + N^iN^j\lambda\partial_iN\partial_j\varphi N^{(-3)} - \\
& 2N^iS\varphi'\partial_i\varphi N^{-1}\gamma^{\frac{1}{2}} - \frac{1}{2}N^i\gamma^{jk}\lambda\gamma_{jk}'\partial_i\varphi N^{(-2)} - \frac{1}{2}N^i\gamma^{jk}\lambda\varphi'\partial_i\gamma_{jk}N^{(-2)} - \\
& \frac{1}{2}K^{ij}K^{kl}N\gamma_{ij}\gamma_{kl}\gamma^{\frac{1}{2}}\kappa^{-1} + \frac{1}{2}K^{ij}K^{kl}N\gamma_{ik}\gamma_{jl}\gamma^{\frac{1}{2}}\kappa^{-1} + N^iN^jS\partial_i\varphi\partial_j\varphi N^{-1}\gamma^{\frac{1}{2}} + \\
& \frac{1}{2}N^iN^j\gamma^{kl}\lambda\partial_i\gamma_{kl}\partial_j\varphi N^{(-2)}). \tag{5.4}
\end{aligned}$$

Momenta corresponding to the variables are defined as

$$\begin{aligned}
\pi^{ij} &= \frac{\delta\mathcal{S}_G}{\delta\gamma_{ij}'} & \pi_N &= \frac{\delta\mathcal{S}_G}{\delta N'} & \pi_i &= \frac{\delta\mathcal{S}_G}{\delta N^{i'}} \\
\pi_\varphi &= \frac{\delta\mathcal{S}_G}{\delta\varphi'} & \pi_\lambda &= \frac{\delta\mathcal{S}_G}{\delta\lambda'} & \pi_S &= \frac{\delta\mathcal{S}_G}{\delta S'}
\end{aligned}$$

Using the action (5.4), we obtain the following relations:

$$\pi^{ij} = \frac{1}{2}\kappa^{-1}\gamma^{\frac{1}{2}}(\gamma^{mn}\gamma^{kl} - \gamma^{mk}\gamma^{nl})K_{kl} + \frac{1}{2}\gamma^{mn}\lambda\varphi'N^{(-2)} - \frac{1}{2}N^i\partial_i\varphi\lambda N^{(-2)}\gamma^{mn} \tag{5.5}$$

$$\begin{aligned}
\pi_\varphi &= -\lambda'N^{(-2)} + N^i\partial_i\lambda N^{(-2)} + 2SN^{-1}\gamma^{\frac{1}{2}}\varphi' + \lambda N'N^{(-3)} - N^i\lambda\partial_iN N^{(-3)} \\
&+ \frac{1}{2}\gamma^{ij}\lambda\gamma_{ij}'N^{(-2)} - 2N^iS\partial_i\varphi N^{-1}\gamma^{\frac{1}{2}} - \frac{1}{2}N^i\gamma^{jk}\lambda\partial_i\gamma_{jk}N^{(-2)} \tag{5.6}
\end{aligned}$$

$$\pi_\lambda = -\varphi'N^{(-2)} + N^i\partial_i\varphi N^{(-2)} \tag{5.7}$$

$$\begin{aligned}
\pi_N &= \varphi'\lambda N^{(-3)} - N^i\lambda\partial_i\varphi N^{(-3)} \\
&= -\lambda N^{-1}\pi_\lambda \tag{5.8}
\end{aligned}$$

$$\pi_i = 0 \tag{5.9}$$

$$\pi_S = 0. \tag{5.10}$$

Equations (5.5), (5.6) and (5.7) are invertible and  $\partial_0\gamma_{ij}$ ,  $\partial_0\lambda$  and  $\partial_0\varphi$  can be written in terms of  $\pi^{ij}$ ,  $\pi_\varphi$  and  $\pi_\lambda$  as

$$\gamma_{mn}' = \gamma_{nk}N^k|_m + \gamma_{mk}N^k|_n - 2NK_{mn} \tag{5.11}$$

$$K_{ij} = \kappa\gamma^{-1/2}(\gamma_{ij}\gamma_{mn} - 2\gamma_{im}\gamma_{jn})\pi^{mn} + \frac{1}{2}\gamma_{ij}\lambda\pi_\lambda \tag{5.12}$$

$$\varphi' = N^i\partial_i\varphi - N^2\pi_\lambda \tag{5.13}$$

$$\begin{aligned}
\lambda' &= N^2(-\pi_\varphi + 2SN^{-1}\gamma^{1/2}\varphi' - 2N^iS\partial_i\varphi N^{-1}\gamma^{1/2} + N^i\partial_i\lambda N^{(-2)} + \lambda N'N^{(-3)} \\
&- N^i\lambda\partial_iN N^{(-3)} + \frac{1}{2}\gamma^{ij}\lambda\gamma_{ij}'N^{(-2)} - \frac{1}{2}N^i\gamma^{jk}\lambda\partial_i\gamma_{jk}N^{(-2)}). \tag{5.14}
\end{aligned}$$

Hence the Hamiltonian density is given by

$$\mathcal{H}_G = \pi^{ij} \gamma'_{ij} + \pi_\varphi \varphi' + \pi_\lambda \lambda' + \pi_N N' - \mathcal{L}_G. \quad (5.15)$$

Using the action (5.4) and (5.11), (5.12), (5.13) (5.14) and (5.8), the Hamiltonian density becomes

$$\begin{aligned} \mathcal{H}_G = & -S\lambda + NV\gamma^{\frac{1}{2}} + N^i \pi_\lambda \partial_i \lambda + N^i \pi_\varphi \partial_i \varphi - \pi_\lambda \pi_\varphi N^2 + \pi_\lambda \lambda \partial_i N^i + 2\gamma_{ij} \partial_k N^i \pi^{jk} + \\ & \gamma^{ij} \lambda \partial_i N \partial_j \varphi N^{-1} - \gamma^{ij} \partial_i \lambda \partial_j \varphi - \lambda \partial_i \gamma^{ij} \partial_j \varphi - \frac{1}{2} N^{(3)} R \gamma^{\frac{1}{2}} \kappa^{-1} - S N^3 \pi_\lambda^2 \gamma^{\frac{1}{2}} - \\ & \frac{3}{4} N \kappa \pi_\lambda^2 \gamma^{-\frac{1}{2}} \lambda^2 - N^i \pi_\lambda \lambda \partial_i N N^{-1} + N^i \partial_i \gamma_{lm} \pi^{lm} - N^i \partial_l \gamma_{im} \pi^{lm} + \\ & N^i \partial_m \gamma_{il} \pi^{lm} - \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{jk} \partial_l \varphi + \frac{1}{2} \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{kl} \partial_j \varphi + N S \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} - \\ & N \pi_\lambda \gamma_{ij} \kappa \lambda \pi^{ij} \gamma^{-\frac{1}{2}} + 2 N \gamma_{ij} \gamma_{kl} \kappa \pi^{ik} \pi^{jl} \gamma^{-\frac{1}{2}} - N \gamma^{ij} \gamma^{kl} \kappa \pi^{ij} \pi^{kl} \gamma^{-\frac{1}{2}}. \end{aligned} \quad (5.16)$$

Since  $\pi_N$  does not appear in the Hamiltonian, complete dynamics of the fields are obtained from the Dirac Hamiltonian

$$\mathcal{H}_D = \mathcal{H}_G + \xi \left( \pi_N + \frac{\lambda}{N} \pi_\lambda \right). \quad (5.17)$$

The above Dirac-Hamiltonian can be used to obtain the perturbed equations of motion.

### 5.1.1 Background equations

At zeroth order, field variables are defined as

$$\begin{aligned} N = N_0, \quad N^i = 0, \quad \gamma_{ij} = a^2 \delta_{ij}, \quad S = S_0, \quad \lambda = \lambda_0, \quad \varphi = \varphi_0 \\ \pi^{ij} = \pi_0^{ij}, \quad \pi_N = \pi_{N0}, \quad \pi_\varphi = \pi_{\varphi 0}, \quad \pi_\lambda = \pi_{\lambda 0}. \end{aligned} \quad (5.18)$$

Using the above relations, the Dirac-Hamiltonian (5.17) takes the form

$$\begin{aligned} \mathcal{H}_{D0} = & -S_0 \lambda_0 + N_0 V_0 a^3 - \pi_{\lambda 0} \pi_{\varphi 0} N_0^2 - S_0 N_0^3 \pi_{\lambda 0}^2 a^3 - \frac{3}{4} N_0 \kappa \pi_{\lambda 0}^2 \lambda_0^2 a^{(-3)} - \\ & \frac{1}{2} N_0 \pi_a \pi_{\lambda 0} \kappa \lambda_0 a^{(-2)} - \frac{1}{12} N_0 \kappa \pi_a^2 a^{-1} + \xi_0 (\pi_{N0} + \pi_{\lambda 0} \lambda_0 N_0^{-1}). \end{aligned} \quad (5.19)$$

Since the momentum corresponding to  $S$  does not appear in the Hamiltonian (5.19), variation of the Hamiltonian with respect to  $S$  leads to the first secondary constraint and it is given by

$$\lambda_0 = -N_0^3 \pi_{\lambda 0}^2 a^3 \quad (5.20)$$

and varying the Dirac-Hamiltonian with respect to  $\xi_0$  recovers the primary constraint

$$\pi_{N0} = -\lambda_0 \pi_{\lambda 0} N_0^{-1}. \quad (5.21)$$

The Hamilton's equations which relate time variation of the field variables with momenta, are given by

$$\varphi_0' = -\pi_{\lambda 0} N_0^2 \quad (5.22)$$

$$a' = -\frac{1}{6} N_0 \kappa a^{-1} \pi_a - \frac{1}{2} N_0 \pi_{\lambda 0} \kappa \lambda_0 a^{(-2)} \quad (5.23)$$

$$N_0' = \xi_0 \quad (5.24)$$

$$\lambda_0' = -\pi_{\varphi 0} N_0^2 - 2 S_0 N_0^3 \pi_{\lambda 0} a^3 - \frac{3}{2} N_0 \kappa \pi_{\lambda 0} \lambda_0^2 a^{(-3)} - \frac{1}{2} N_0 \pi_a \kappa \lambda_0 a^{(-2)} + \lambda_0 N_0^{-1} \xi_0 \quad (5.25)$$

$$(5.26)$$

which relate the momenta and time derivatives of the fields. The above relations are invertible and all momenta can be written in terms of the time derivatives of the field variables. Hamilton's equations corresponding to the time variation of momenta are given by:

$$\begin{aligned} \pi_a' &= -3 N_0 V_0 a^2 + 3 S_0 N_0^3 \pi_{\lambda 0}^2 a^2 - \frac{9}{4} N_0 \kappa \pi_{\lambda 0}^2 \lambda_0^2 a^{(-4)} - N_0 \pi_a \pi_{\lambda 0} \kappa \lambda_0 a^{(-3)} \\ &\quad - \frac{1}{12} N_0 \kappa \pi_a^2 a^{(-2)} \end{aligned} \quad (5.27)$$

$$\pi_{\lambda 0}' = S_0 - 3 \pi_{\lambda 0} a' a^{-1} - \pi_{\lambda 0} \partial_0 N_0 N_0^{-1} \quad (5.28)$$

$$\begin{aligned} \pi_{N_0}' &= -V_0 a^3 + 2 \pi_{\lambda 0} \pi_{\varphi 0} N_0 + 3 S_0 N_0^2 \pi_{\lambda 0}^2 a^3 + \frac{3}{4} \kappa \pi_{\lambda 0}^2 \lambda_0^2 a^{(-3)} \\ &\quad + \frac{1}{2} \pi_a \pi_{\lambda 0} \kappa \lambda_0 a^{(-2)} + \frac{1}{12} \kappa \pi_a^2 a^{-1} + \pi_{\lambda 0} \lambda_0 N_0^{(-2)} \xi_0 \end{aligned} \quad (5.29)$$

$$\pi_{\varphi 0}' = -N_0 V_{\varphi} a^3. \quad (5.30)$$

which, by using other background Hamilton's equations, lead to the identical dynamical equations of motion of the field variables obtained from action formulation. Using background Hamilton's equations in conformal time coordinate, equation (5.28) becomes

$$S_0 = -2H \varphi_0' a^{(-2)} - \varphi_0'' a^{(-2)}. \quad (5.31)$$

Variation of action (5.2) or (5.3) with respect to  $\lambda$  lead to  $S = \square\varphi$ . At zeroth order,  $\square\varphi = -2H \varphi_0' a^{(-2)} - \varphi_0'' a^{(-2)}$  implying that the dynamical equation (5.31) obtained using Hamiltonian formulation is consistent. Similarly, equation (5.29) leads to the zeroth order Hamiltonian constraint of the Galilean scalar field model. In conformal time, it is given by

$$V_0 a^2 - 6 H a^{(-2)} \varphi_0'^3 - 3 \kappa^{-1} H^2 = 0 \quad (5.32)$$

Equations (5.27) and (5.30) lead to the equation of motion of  $a$  and  $\varphi_0$  respectively and are given by

$$6 H \varphi_0'^3 a^{(-2)} - 6 \varphi_0'' \varphi_0'^2 a^{-2} + 3 \kappa^{-1} H^2 - 6 a'' a^{-1} \kappa^{-1} + 3 V_0 a^2 = 0 \quad (5.33)$$

$$\frac{1}{2} \varphi_0'^2 a'' a^{-1} H^{-1} - \frac{H}{2} \varphi_0'^2 + \varphi_0'' \varphi_0' - \frac{1}{12} V_{\varphi} H^{-1} a^4 = 0. \quad (5.34)$$

Equations (5.32), (5.33) and (5.34) are identical to the Lagrangian equations (C.3), (C.4) and (C.5), respectively. Hence, at zeroth order, Hamiltonian formulation is consistent with Lagrangian formulation.

**Counting scalar degrees of freedom at zeroth order:** As one can see in (5.19), background phase space contains 10 variable ( $a, N, \varphi_0, \lambda_0, S_0$  and corresponding momenta). There are two primary constrained equations:

$$\Phi_p^1 \equiv \pi_{S_0} = 0 \quad (5.35)$$

$$\Phi_p^2 \equiv \pi_{N_0} + \lambda_0 \pi_{\lambda_0} N_0^{-1} = 0 \quad (5.36)$$

Conservation of primary constraints gives rise to secondary constraints:

$$\Phi_s^1 \equiv \{\Phi_p^1, \mathcal{H}_{D0}\} \approx 0 \Rightarrow \lambda_0 + N_0^3 \pi_{\lambda_0}^2 a^3 \approx 0 \quad (5.37)$$

$$\begin{aligned} \Phi_s^2 \equiv \{\Phi_p^2, \mathcal{H}_{D0}\} \approx 0 \Rightarrow & -V_0 a^3 + \pi_{\lambda_0} \pi_{\varphi_0} N_0 + \frac{3}{4a^3} \kappa \pi_{\lambda_0}^2 \lambda_0^2 + \frac{1}{12a} \kappa \pi_a^2 \\ & + \frac{\kappa}{2a^2} N_0 \pi_a \pi_{\lambda_0} \lambda_0 = 0 \end{aligned} \quad (5.38)$$

Equation (5.38) leads to zeroth order Hamiltonian constraint and is equivalent to equation (C.2). Further, conservation of secondary constraint (5.37) leads to tertiary constraint

$$\Phi_t \equiv \{\Phi_s^1, \mathcal{H}_{D0}\} \approx 0 \Rightarrow \pi_{\varphi_0} - \kappa N_0^2 \pi_{\lambda_0}^2 a - 3\kappa N_0^2 \pi_{\lambda_0}^3 \lambda_0 = 0 \quad (5.39)$$

and generates quaternary constraint

$$\Phi_q \equiv \{\Phi_t, \mathcal{H}_{D0}\} \approx 0 \quad (5.40)$$

$$\begin{aligned} \Rightarrow S_0 (-2\kappa N_0^2 \pi_{\lambda_0} a + 15\kappa N_0^5 \pi_{\lambda_0}^5 \pi_a a) - N_0 V_\varphi - \frac{5}{2} \kappa^2 N_0^3 \pi_{\lambda_0}^3 \lambda_0 a^{-2} \\ + \frac{5}{6} \kappa^2 N_0^3 \pi_{\lambda_0}^2 \pi_a a^{-1} + 18\kappa^2 N_0^6 \pi_{\lambda_0}^6 \lambda_0 + 6\kappa^2 N_0^6 \pi_{\lambda_0}^6 \pi_a a = 0 \end{aligned} \quad (5.41)$$

Out of the 6 constrained equations, equations (5.36) and (5.38) are first class and rest are second class constraints. Hence, in coordinate space, number of degrees of freedom is

$$\frac{1}{2} \times (10 - 2 \times 2 - 4) = 1$$

which is same as canonical scalar field model.

### 5.1.2 First order perturbed equations

The first order perturbation of the field variables and the momenta are defined as

$$\begin{aligned} N &= N_0 + \epsilon N_1, & N^i &= \epsilon N_1^i, & \gamma_{ij} &= a^2 \delta_{ij}, & S &= S_0 + \epsilon S_1, \\ \lambda &= \lambda_0 + \epsilon \lambda_1, & \varphi &= \varphi_0 + \epsilon \varphi_1, & \pi^{ij} &= \pi_0^{ij} + \epsilon \pi_1^{ij}, & \pi_N &= \pi_{N_0} + \epsilon \pi_{N_1}, \\ \pi_\varphi &= \pi_{\varphi_0} + \epsilon \pi_{\varphi_1}, & \pi_\lambda &= \pi_{\lambda_0} + \epsilon \pi_{\lambda_1}. \end{aligned} \quad (5.42)$$

First order perturbed Hamiltonian equations are obtained by varying the second order perturbed Hamiltonian. Using above definitions of perturbations in the Hamiltonian (5.17),

the second order perturbed Hamiltonian becomes

$$\begin{aligned}
\mathcal{H}_{D2} = & -S_1\lambda_1 + N_1V_\varphi a^3\varphi_1 + \frac{1}{2}N_0V_{\varphi\varphi}\varphi_1^2a^3 + N_1^i\pi_{\lambda 0}\partial_i\lambda_1 + N_1^i\pi_{\varphi 0}\partial_i\varphi_1 - \\
& 2N_0N_1\pi_{\lambda 0}\pi_{\varphi 1} - 2N_0N_1\pi_{\lambda 1}\pi_{\varphi 0} - \pi_{\lambda 1}\pi_{\varphi 1}N_0^2 + \pi_{\lambda 1}\lambda_0\partial_iN_1^i + \pi_{\lambda 0}\lambda_1\partial_iN_1^i - \\
& \delta^{ij}\partial_i\lambda_1\partial_j\varphi_1a^{(-2)} - S_0N_0^3\pi_{\lambda 1}^2a^3 - 6N_1\pi_{\lambda 0}\pi_{\lambda 1}S_0N_0^2a^3 - 3N_0S_0N_1^2\pi_{\lambda 0}^2a^3 - \\
& 2\pi_{\lambda 0}\pi_{\lambda 1}S_1N_0^3a^3 - 3N_1S_1N_0^2\pi_{\lambda 0}^2a^3 - \frac{3}{4}N_0\kappa\pi_{\lambda 0}^2\lambda_1^2a^{(-3)} - \\
& \frac{3}{4}N_0\kappa\pi_{\lambda 1}^2\lambda_0^2a^{(-3)} - \frac{3}{2}N_1\kappa\lambda_0\lambda_1\pi_{\lambda 0}^2a^{(-3)} - \frac{3}{2}N_1\pi_{\lambda 0}\pi_{\lambda 1}\kappa\lambda_0^2a^{(-3)} + \\
& \delta^{ij}\lambda_0\partial_iN_1\partial_j\varphi_1N_0^{-1}a^{(-2)} + N_0S_0\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a - N_0\pi_{\lambda 1}\delta_{ij}\kappa\lambda_0\pi_1^{ij}a^{-1} - \\
& N_1\pi_{\lambda 0}\delta_{ij}\kappa\lambda_0\pi_1^{ij}a^{-1} - N_1\pi_{\lambda 1}\delta_{ij}\kappa\lambda_0\pi_0^{ij}a^{-1} - N_0\pi_{\lambda 0}\delta_{ij}\kappa\lambda_1\pi_1^{ij}a^{-1} - \\
& N_1\pi_{\lambda 0}\delta_{ij}\kappa\lambda_1\pi_0^{ij}a^{-1} - N_0\delta_{ij}\delta_{kl}\kappa\pi_1^{ij}\pi_1^{kl}a - 2N_1\delta_{ij}\delta_{kl}\kappa\pi_0^{ij}\pi_1^{kl}a + \\
& 4N_1\delta_{ij}\delta_{kl}\kappa\pi_0^{ik}\pi_1^{jl}a + \pi_{N1}\xi_1 - N_1\pi_{\lambda 0}\lambda_0N_0^{(-2)}\xi_1 + \pi_{\lambda 0}\lambda_0N_0^{(-3)}N_1^2\xi_0 - \\
& N_1\pi_{\lambda 1}\lambda_0N_0^{(-2)}\xi_0 + \pi_{\lambda 0}\lambda_1N_0^{-1}\xi_1 - N_1\pi_{\lambda 0}\lambda_1N_0^{(-2)}\xi_0 + \pi_{\lambda 1}\lambda_1N_0^{-1}\xi_0 \\
& - \pi_{\lambda 0}\pi_{\varphi 0}N_1^2 + 2\delta_{ij}\partial_kN_1^i\pi_1^{jk}a^2 - 3N_0\pi_{\lambda 0}\pi_{\lambda 1}\kappa\lambda_0\lambda_1a^{(-3)} + \pi_{\lambda 1}\lambda_0N_0^{-1}\xi_1 \\
& + 2N_0\delta_{ij}\delta_{kl}\kappa\pi_1^{ik}\pi_1^{jl}a - N_0\pi_{\lambda 1}\delta_{ij}\kappa\lambda_1\pi_0^{ij}a^{-1} - N_1^i\pi_{\lambda 0}\lambda_0\partial_iN_1N_0^{-1}. \tag{5.43}
\end{aligned}$$

Equations of motion corresponding to the perturbed Hamiltonian (5.43) is expressed in Appendix 5.1.2.

Variation of the perturbed Hamiltonian density (5.43) with respect to  $S_1$  lead to the secondary constraint

$$\lambda_1 = -2\pi_{\lambda 1}\pi_{\lambda 0}N_0^3a^3 - 3N_1N_0^2\pi_{\lambda 0}^2a^3. \tag{5.44}$$

Variation of (5.43) with respect to  $\xi_1$  leads to the equation

$$\pi_{N1} = -\lambda_0\pi_{\lambda 1}N_0^{-1} - \lambda_1\pi_{\lambda 0}N_0^{-1} + \lambda_0\pi_{\lambda 0}N_0^{(-2)}N_1 \tag{5.45}$$

which constrains  $\pi_{N1}$  with  $\pi_{\lambda 1}$ . Since there is no perturbation in the 3-metric, variation with respect to  $\pi_1^{ij}$ , as expected, is equal to zero and contracting with  $(\delta^{mn}\delta^{ij} - \delta^{mi}\delta^{nj})$ , we get

$$\begin{aligned}
\pi_1^{ij} = & \frac{1}{2}N_0^{-1}a\kappa^{-1}\delta^{ij}\partial_kN_1^k - \frac{1}{2}a^{(-2)}\delta^{ij}\pi_{\lambda 1}\lambda_0 - \frac{1}{2}a^{(-2)}N_1N_0^{-1}\delta^{ij}\pi_{\lambda 0}\lambda_0 \\
& - \frac{1}{2}a^{(-2)}\delta^{ij}\pi_{\lambda 0}\lambda_1 - \frac{1}{4}N_0^{-1}\kappa^{-1}a\delta^{kj}\partial_kN_1^i - \frac{1}{4}N_0^{-1}\kappa^{-1}a\delta^{ki}\partial_kN_1^j - N_0^{-1}N_1\pi_0^{ij} \tag{5.46}
\end{aligned}$$

At first order perturbation, Hamilton's equations corresponding to the time variation of field variables are given by

$$\begin{aligned}
\lambda_1' = & -2N_0N_1\pi_{\varphi 0} - \pi_{\varphi 1}N_0^2 + \lambda_0\partial_iN_1^i - 2\pi_{\lambda 1}S_0N_0^3a^3 - 6N_1\pi_{\lambda 0}S_0N_0^2a^3 - \\
& 2\pi_{\lambda 0}S_1N_0^3a^3 - 3N_0\pi_{\lambda 0}\kappa\lambda_0\lambda_1a^{(-3)} - \frac{3}{2}N_0\pi_{\lambda 1}\kappa\lambda_0^2a^{(-3)} - \frac{3}{2}N_1\pi_{\lambda 0}\kappa\lambda_0^2a^{(-3)} \\
& - N_0\delta_{ij}\kappa\lambda_0\pi_1^{ij}a^{-1} - N_1\delta_{ij}\kappa\lambda_0\pi_0^{ij}a^{-1} - N_0\delta_{ij}\kappa\lambda_1\pi_0^{ij}a^{-1} + \\
& \lambda_0N_0^{-1}\xi_1 - N_1\lambda_0N_0^{(-2)}\xi_0 + \lambda_1N_0^{-1}\xi_0 \tag{5.47}
\end{aligned}$$

$$\varphi_1' = -2N_0N_1\pi_{\lambda 0} - \pi_{\lambda 1}N_0^2 \tag{5.48}$$

$$N_1' = \xi_1 \tag{5.49}$$

Hamilton's equation corresponding to the time variation of the momentum  $\pi_{\lambda 1}$  is given by

$$\pi_{\lambda 1}' + \frac{\delta \mathcal{H}_{D2}}{\delta \lambda_1} = 0 \quad (5.50)$$

$$\begin{aligned} \Rightarrow \pi_{\lambda 1}' - S_1 + \delta^{ij} \partial_{ij} \varphi_1 a^{(-2)} - \frac{3}{2} N_0 \kappa \pi_{\lambda 0}^2 \lambda_1 a^{(-3)} - 3 N_0 \pi_{\lambda 0} \pi_{\lambda 1} \kappa \lambda_0 a^{(-3)} - \\ N_0 \pi_{\lambda 0} \delta_{ij} \kappa \pi_1^{ij} a^{-1} - \frac{3}{2} N_1 \kappa \lambda_0 \pi_{\lambda 0}^2 a^{(-3)} - N_0 \pi_{\lambda 1} \delta_{ij} \kappa \pi_0^{ij} a^{-1} \\ - N_1 \pi_{\lambda 0} \delta_{ij} \kappa \pi_0^{ij} a^{-1} + \pi_{\lambda 0} N_0^{-1} \xi_1 - N_1 \pi_{\lambda 0} N_0^{(-2)} \xi_0 + \pi_{\lambda 1} N_0^{-1} \xi_0 = 0 \end{aligned} \quad (5.51)$$

which, using other first order equations and (4.25), becomes

$$\begin{aligned} -S_1 - \varphi_1'' a^{(-2)} + \delta^{ij} \partial_{ij} \varphi_1 a^{(-2)} + \phi_1' \varphi_0' a^{(-2)} - 2 \varphi_1' a' a^{(-3)} + \\ 2 \varphi_0'' \phi_1 a^{(-2)} + \delta^{ij} \varphi_0' \partial_{ij} B_1 a^{(-2)} + 4 \varphi_0' a' \phi_1 a^{(-3)} = 0 \\ \Rightarrow S_1 = -\varphi_1'' a^{(-2)} + \delta^{ij} \partial_{ij} \varphi_1 a^{(-2)} + \phi_1' \varphi_0' a^{(-2)} - 2 \varphi_1' H a^{(-2)} \\ + 2 \varphi_0'' \phi_1 a^{(-2)} + \delta^{ij} \partial_{ij} B_1 \varphi_0' a^{(-2)} + 4 \varphi_0' H \phi_1 a^{(-2)}. \end{aligned} \quad (5.52)$$

Right hand side of the above equation is the explicit form of first order perturbed  $\square \varphi$ . Hence, the first equation obtained from perturbed Hamiltonian is consistent.

First order perturbed Hamiltonian constraint is obtained by the time variation of  $\pi_{N1}$  and is given by

$$\begin{aligned} \pi_{N1}' + \frac{\delta \mathcal{H}_{D2}}{\delta N_1} = 0 \quad (5.53) \\ \Rightarrow \pi_{N1}' + V_\varphi a^3 \varphi_1 - 2 N_1 \pi_{\lambda 0} \pi_{\varphi 0} - 2 N_0 \pi_{\lambda 0} \pi_{\varphi 1} - 2 N_0 \pi_{\lambda 1} \pi_{\varphi 0} - 6 \pi_{\lambda 0} \pi_{\lambda 1} S_0 N_0^2 a^3 - \\ 6 N_0 N_1 S_0 \pi_{\lambda 0}^2 a^3 - 3 S_1 N_0^2 \pi_{\lambda 0}^2 a^3 - \frac{3}{2} \kappa \lambda_0 \lambda_1 \pi_{\lambda 0}^2 a^{(-3)} - \frac{3}{2} \pi_{\lambda 0} \pi_{\lambda 1} \kappa \lambda_0^2 a^{(-3)} \\ + \pi_{\lambda 0} \lambda_0 \partial_i N_1^i N_0^{-1} - \delta^{ij} \lambda_0 \partial_{ij} \varphi_1 N_0^{-1} a^{(-2)} - \pi_{\lambda 0} \delta_{ij} \kappa \lambda_0 \pi_1^{ij} a^{-1} - \\ \pi_{\lambda 1} \delta_{ij} \kappa \lambda_0 \pi_0^{ij} a^{-1} - \pi_{\lambda 0} \delta_{ij} \kappa \lambda_1 \pi_0^{ij} a^{-1} - 2 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_1^{kl} a + 4 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_1^{jl} a \\ - \pi_{\lambda 0} \lambda_0 N_0^{(-2)} \xi_1 + 2 N_1 \pi_{\lambda 0} \lambda_0 N_0^{(-3)} \xi_0 - \pi_{\lambda 1} \lambda_0 N_0^{(-2)} \xi_0 - \pi_{\lambda 0} \lambda_1 N_0^{(-2)} \xi_0 = 0 \end{aligned} \quad (5.54)$$

In flat-slicing gauge, it becomes

$$\begin{aligned} \mathcal{H}_{N1} \equiv 24 H \phi_1 \varphi_0'^3 a^{(-2)} - 18 \varphi_1' H \varphi_0'^2 a^{(-2)} + V_\varphi a^2 \varphi_1 + 2 \delta^{ij} \partial_{ij} B_1 \varphi_0'^3 a^{-2} + \\ 2 \delta^{ij} \partial_{ij} \varphi_1 \varphi_0'^2 a^{-2} + 2 \delta^{ij} H \partial_{ij} B_1 \kappa^{-1} + 6 \phi_1 \kappa^{-1} H^2 = 0. \end{aligned} \quad (5.55)$$

Since there is no momentum  $\pi_i^1$  corresponding to  $N_1^i$  appeared in the second order Hamiltonian (5.43), variation of the Hamiltonian with respect to  $N_1^i$  leads to Momentum constraint of Galilean field

$$M_{i1} \equiv \pi_{\varphi 0} \partial_i \varphi_1 - \lambda_0 \partial_i \pi_{\lambda 1} - 2 \delta_{ij} a^2 \partial_k \pi_1^{jk} - \pi_{\lambda 0} \lambda_0 \partial_i N_1 N_0^{-1} = 0, \quad (5.56)$$

which, in flat-slicing gauge, becomes

$$M_{i1} \equiv -6 H \partial_i \varphi_1 \varphi_0'^2 - 2 \partial_i \phi_1 \varphi_0'^3 + 2 \partial_i \varphi_1' \varphi_0'^2 - 2 H \partial_i \phi_1 \kappa^{-1} a^2 = 0. \quad (5.57)$$

Similarly, equation of motion of  $\varphi_1$  can be obtained from

$$\pi_{\varphi 1}' + \frac{\delta \mathcal{H}_{D2}}{\delta \varphi_1} = 0 \quad (5.58)$$

$$\begin{aligned} \Rightarrow \pi_{\varphi 1}' + N_1 V_\varphi a^3 + N_0 V_{\varphi \varphi} \varphi_1 a^3 - \partial_i N_1^i \pi_{\varphi 0} + \delta^{ij} \partial_{ij} \lambda_1 a^{(-2)} \\ - \delta^{ij} \lambda_0 \partial_{ij} N_1 N_0^{-1} a^{(-2)} - 2 N_0 S_0 \delta^{ij} \partial_{ij} \varphi_1 a = 0. \end{aligned} \quad (5.59)$$

Using (4.25), the above equation can be written as

$$\begin{aligned}
& -18 \phi_1 \phi_0'^2 H^2 + 12 \phi_0' \phi_1' H^2 + 18 \phi_1' H \phi_0'^2 \\
& + 36 \phi_0' H \phi_0'' \phi_1 - 12 \phi_0' H \phi_1'' - 12 \phi_1' H \phi_0'' \\
& - 12 \phi_0' \phi_1' a'' a^{-1} + 6 \delta^{ij} H \partial_{ij} B_1 \phi_0'^2 + 4 \delta^{ij} \phi_0' \phi_0'' \partial_{ij} B_1 + \\
& 4 \delta^{ij} \phi_0' H \partial_{ij} \phi_1 a^{-1} + 4 \delta^{ij} \phi_0'' \partial_{ij} \phi_1 + 2 \delta^{ij} \partial_{ij} B_1 \phi_0'^2 + V_\varphi \phi_1 a^4 + \\
& V_{\varphi\varphi} a^4 \phi_1 + 2 \delta^{ij} \partial_{ij} \phi_1 \phi_0'^2 + 18 a'' \phi_1 \phi_0'^2 a^{-1} = 0.
\end{aligned} \tag{5.60}$$

Equations (5.55), (5.57) and (5.60) obtained from Hamiltonian formulation are identical to (C.6), (C.7) and (C.8), respectively.

Third and fourth order interaction Hamiltonian are given in Appendix D.

**Counting scalar degrees of freedom at first order:** Since we are considering only scalar first order perturbations,  $N_1^i$  contains one scalar variable ( $N_1^i = \delta^{ij} \partial_j B_1$ ) and hence,  $\pi_i = 0$  and Momentum constraint (5.56) lead to two constrained equations. Along with (5.45) and Hamiltonian constraint (5.55), we get 4 constrained equations. Similarly, as we have seen in zeroth order, at first order, we get four second class constraints:

$$\Phi_p \equiv \pi_S = 0 \tag{5.61}$$

$$\Phi_s \equiv \{\pi_S, \mathcal{H}_{D2}\} = \lambda_1 + 2 \pi_{\lambda 1} \pi_{\lambda 0} N_0^3 a^3 + 3 N_1 N_0^2 \pi_{\lambda 0}^2 a^3 = 0 \tag{5.62}$$

$$\begin{aligned}
\Phi_t \equiv \{\Phi_s, \mathcal{H}_{D2}\} = & \pi_{\varphi 1} + 2 N_1 \pi_{\varphi 0} N_0^{-1} - 2 N_0 \partial_i N_1^i \pi_{\lambda 0}^2 a^3 + 2 N_0 \pi_{\lambda 0} \delta^{ij} \partial_{ij} \phi_1 a + \\
& 9 N_1 \kappa N_0^4 \pi_{\lambda 0}^5 a^3 + 6 \pi_{\lambda 1} \kappa N_0^5 \pi_{\lambda 0}^4 a^3 - 3 N_0 N_1 \pi_a \kappa \pi_{\lambda 0}^2 a \\
& - 2 \pi_a \pi_{\lambda 0} \pi_{\lambda 1} \kappa N_0^2 a = 0
\end{aligned} \tag{5.63}$$

$$\Phi_q \equiv \{\Phi_t, \mathcal{H}_{D2}\} \approx 0 \tag{5.64}$$

We also get 12 more second class constraints:  $\delta\gamma_{ij} = 0$  and  $\{\delta\gamma_{ij},^{(2)}\mathcal{H}_D\}$  which leads to equation (5.46). Since, we have fixed the gauge, all constraints become second class. Our Galilean phase space contains 22 variables. Hence in configuration space, the number of degrees of freedom is

$$\frac{1}{2} \times (22 - 4 - 4 - 12) = 1.$$

This procedure can be extended to higher order and at any order it can be shown that the degrees of freedom is one. So, at any order, Galilean scalar field produces no extra degrees of freedom due to the higher derivative terms present in the Lagrangian and behave exactly same as any single derivative Lagrangian system.

However, if we consider generalized Lagrangian containing second order derivative terms, the above analysis can not be extended. In that case, unlike Galilean field, the Lapse function and Shift vector will not act like constraints and hence,  $\pi_N + \dots \neq 0$ ,  $\pi_i + \dots \neq 0$  since  $\pi_N$  and  $\pi_i$  contain time derivatives of  $N$  and  $N^i$  [48], and dynamical degrees of freedom can be generated from those. This means that for any higher derivative gravitational theory, lack of first class constraints lead to extra degrees of freedom. Similarly, extra degrees of freedom will always be generated for any generalized second order derivative Lagrangian.

## 6 Conclusion and Discussion

In this work, using Hamiltonian formulation, we have formulated a consistent cosmological perturbation theory at all orders. We have adopted the following procedures: we choose

a particular gauge that does not lead to any particular gauge artifact[52] such that some variables remain unperturbed while others can be separated as zeroth order part and perturbation part. In order to make the procedure transparent, we considered a simple model of two variables where one variable is unperturbed and other variable can be perturbed. At first order, we confronted the gauge-issue and found that, even canonical conjugate momentum of unperturbed quantity has perturbation part that leads a constrained equation at every perturbed order and by using the equation we can get the exact form of perturbed momentum. We fixed the gauge-issue and obtained all first order perturbed equations as well as third and fourth order perturbed Hamiltonian which is consistent with Lagrangian formulation. The procedure is simple and robust and can be extended to any order of perturbation. Table below provides a bird's eye view of the both the formulations and advantages of the Hamiltonian formulation that is proposed in this work:

	Lagrangian formulation	Hamiltonian formulation
Gauge conditions and gauge-invariant equations	At any order, choose a gauge which does not lead to gauge-artifacts	Choose a gauge with no gauge-artifacts, however, momentum corresponding to unperturbed quantity is non-zero leading to consistent equations of motion.
Dynamical variables	Counting true dynamical degrees of freedom is difficult.	Using Dirac's procedure, constraints can easily be obtained and is easy to determine the degrees of freedom.
Quantization at all orders	Difficult to quantize constrained system.	Since constraints are obtained systematically and reduced phase space contains only true degrees of freedom, it is straightforward to quantize the theory using Hamiltonian formulation.
Calculating the observables	Requires to invert the expressions at each order and hence non-trivial to compute higher-order correlation function.	Once the relation between $\varphi$ and Curvature perturbation <sup>4</sup> is known, calculating the correlation functions from the Hamiltonian is simple and straightforward to obtain.

We have applied the procedure to canonical scalar field minimally coupled to gravity and similarly obtained all equations as well as interaction Hamiltonian using both Lagrangian and Hamiltonian formulation. Both lead to identical results. We also showed that, obtaining interaction Hamiltonian by using Hamiltonian formulation is efficient and straightforward. Unlike the Lagrangian formulation, we do not need to invert the expressions at each order [43].

We, then obtained a consistent perturbed Hamiltonian formulation for Galilean scalar fields. Using flat-slicing gauge, we have obtained zeroth and first order Hamilton's equations that are consistent with Lagrangian formulation. We carefully analyzed the constraints in the system and counted degrees of freedom at every order in the system that is consistent with the results of Deffayet *et al*[44] results. It has been shown that general higher derivative models lead to dynamical equations of Lapse function/shift vector which increases the number

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<sup>4</sup>It is important to note that, in the case of first order, relation between  $\varphi$  and three-curvature is straightforward. However, it is more subtle in the case of higher-order perturbations[52].



degrees of freedom of the system. But, only in higher derivative Galilean theory, Lapse function/shift vector remain constraint, therefore, no extra degrees of freedom flows in the system. Similar type of problem has been encountered in a different manner in Ref. [48].

To make the Physics transparent, in this work, we have neglected vector and tensor perturbations. Our approach can be applied to higher-order perturbations including vector and tensor perturbations. In the presence of tensor or vector or any mixed modes at any higher perturbed order, since modes do not decouple,  $\pi^{ij}$  cannot decouple and act as the momentum corresponding the overall 3-metric  $\delta\gamma_{ij}$  which contains mixed modes. Hence,

$$\begin{aligned}\frac{\delta\mathcal{H}}{\delta\pi^{ij}} &= \partial_0(\delta\gamma_{ij}) \\ &= \partial_0(\delta\gamma_{ij}^S + \delta\gamma_{ij}^V + \delta\gamma_{ij}^T),\end{aligned}$$

where  $\gamma_{ij}^S, \gamma_{ij}^V, \gamma_{ij}^T$  are scalar, vector and tensor modes, respectively.

Our approach can be applied to any model of gravity and matter fields to obtain any higher order interaction Hamiltonian without invoking any approximation such as slow roll, etc. It can also been shown that, the mechanism can be applied for any generalized tensor fields and it can even extract any higher order cross-correlation interaction Hamiltonian.

Our approach can also be used for modified gravity models including  $f(R)$  model, other scalar-tensor theories like Gauss-Bonnet inflation, Lovelock gravity, Hordenski theory. In fact, we can say unequivocally that our approach can be used for any kind of gravity models.

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## A Perturbed equations of motion of Canonical scalar field in flat-slicing gauge

### A.1 Background equations

0-0 component of the Einstein's equation or the Hamiltonian constraint in conformal coordinate is given by

$$H^2 \equiv a'^2 a^{-2} = \frac{\kappa}{3} \left[ \frac{1}{2} \varphi_0'^2 + V a^2 \right]. \quad (\text{A.1})$$

Trace of i-j component of the Einstein's equation gives the equation of motion of  $a$ , which can also be obtained by varying the zeroth order action with respect to  $a$  and is given by

$$3\kappa^{-1}H^2 - 6\frac{a''}{a}\kappa^{-1} - \frac{3}{2}\varphi_0'^2 + 3Va^2 = 0 \quad (\text{A.2})$$

and the equation of motion of scalar field at zeroth order takes the form

$$\varphi_0'' + 2H\varphi_0' + V_\varphi a^2 = 0. \quad (\text{A.3})$$

## A.2 First order perturbed equations

Using the perturbed metric and perturbed scalar field defined in (2.12), (2.13), (2.14) and (2.15), perturbed Hamiltonian constraint or the first order perturbed 0-0 component of the Einstein's equation becomes

$$2\delta^{ij}H\partial_{ij}B_1\kappa^{-1} + 6\phi_1\kappa^{-1}H^2 + \varphi'_0\varphi'_1 - \phi_1\varphi'_0{}^2 + V_\varphi a^2\varphi_1 = 0. \quad (\text{A.4})$$

Similarly, perturbed Momentum constraint or perturbed 0-i component of the Einstein's equation is

$$\frac{2H}{\kappa}\partial_i\phi_1 = \varphi'_0\partial_i\varphi_1 \quad (\text{A.5})$$

$$\Rightarrow \phi_1 = \frac{\kappa}{2H}\varphi'_0\varphi_1 \quad (\text{A.6})$$

and equation of motion of the scalar field  $\varphi_1$ , using (A.3) is given by

$$\varphi_1'' + 2H\varphi_1' - \phi_1'\varphi_0' + 2V_\varphi\phi_1a^2 - \delta^{ij}\varphi_0'\partial_{ij}B_1 - \delta^{ij}\partial_{ij}\varphi_1 + V_{\varphi\varphi}a^2\varphi_1 = 0. \quad (\text{A.7})$$

## A.3 Third order interaction Hamiltonian of Canonical scalar field in terms of phase-space variables in flat-slicing gauge

Third or higher order perturbed Hamiltonian in terms of first order perturbed variables is needed to calculate the interaction Hamiltonian which helps to calculate higher order correlation functions. The third order Hamiltonian is obtained by expanding the Hamiltonian (4.8) up to third order and is given by

$$\begin{aligned} \mathcal{H}_3^C(\pi, \varphi) = & -\frac{1}{2}N_1\partial_iN_1{}^i\partial_jN_1{}^jN_0^{(-2)}\kappa^{-1}a^3 + 2\delta_{ij}\partial_kN_1{}^k\pi_0{}^{ij}N_0^{(-2)}N_1{}^2a^2 - \\ & 2\delta_{ij}\delta_{kl}\kappa\pi_0{}^{ij}\pi_0{}^{kl}N_0^{(-2)}N_1{}^3a + \frac{1}{4}N_1\delta_{ij}\delta^{lk}\partial_kN_1{}^i\partial_lN_1{}^jN_0^{(-2)}\kappa^{-1}a^3 + \\ & \frac{1}{4}N_1\partial_iN_1{}^j\partial_jN_1{}^iN_0^{(-2)}\kappa^{-1}a^3 + \frac{1}{2}\delta_{ij}\partial_kN_1{}^i\pi_0{}^{kj}N_0^{(-2)}N_1{}^2a^2 + \\ & \frac{1}{2}\delta_{ij}\partial_kN_1{}^i\pi_0{}^{jk}N_0^{(-2)}N_1{}^2a^2 - 2\delta_{ij}\partial_kN_1{}^k\pi_0{}^{ij}N_0^{(-2)}N_1{}^2a^2 + \\ & \frac{1}{2}\delta_{ij}\partial_kN_1{}^j\pi_0{}^{ki}N_0^{(-2)}N_1{}^2a^2 + \frac{1}{2}\delta_{ij}\partial_kN_1{}^j\pi_0{}^{ik}N_0^{(-2)}N_1{}^2a^2 + \\ & 2\delta_{ij}\delta_{kl}\kappa\pi_0{}^{ik}\pi_0{}^{jl}N_0^{(-2)}N_1{}^3a + \frac{1}{2}N_1\pi_{\varphi_1}{}^2a^{(-3)} + N_1{}^i\pi_{\varphi_1}\partial_i\varphi_1 + \\ & \delta_{ij}\delta_{kl}\kappa\pi_0{}^{ij}\pi_0{}^{lk}N_0^{(-2)}N_1{}^3a + \frac{1}{2}N_1\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a + \frac{1}{2}N_1V_{\varphi\varphi}\varphi_1{}^2a^3 + \\ & \frac{1}{6}N_0V_{\varphi\varphi\varphi}\varphi_1{}^3a^3 \end{aligned} \quad (\text{A.8})$$

## B Hamiltonian formulation of Canonical scalar field in uniform density gauge

In uniform-density gauge,  $E = \delta\varphi = 0$  and  $\gamma_{ij} = a^2(1 - 2\epsilon\psi_1)\delta_{ij}$ ,  $\gamma^{ij} = a^2(1 + 2\epsilon\psi_1 + 4\epsilon^2\psi_1^2)\delta^{ij}$ ,  $\sqrt{\gamma} = a^6(1 - 6\epsilon\psi_1 + 12\epsilon^2\psi_1^2)$ . The second order perturbed Hamiltonian is obtained

by expanding the Hamiltonian (4.8) up to second order by using above definitions and is given by

$$\begin{aligned}
\mathcal{H}_2^C = & \delta_{ij} \partial_k N_1^j \pi_1^{ik} a^2 - 2 \delta_{ij} \partial_k N_1^j \pi_0^{ik} a^2 \psi_1 - 2 N_1^i \delta_{jk} \partial_i \psi_1 \pi_0^{jk} a^2 + \\
& \delta_{ij} \partial_k N_1^j \pi_1^{ik} a^2 - 2 \delta_{ij} \partial_k N_1^j \pi_0^{ik} a^2 \psi_1 - N_0 \delta_{ij} \delta_{kl} \kappa \pi_1^{ij} \pi_1^{kl} a - \\
& 2 N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_1^{kl} a + 2 N_0 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_1^{kl} \psi_1 a + N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_0^{kl} \psi_1 a + \\
& \frac{1}{2} N_0 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_0^{kl} \psi_1^2 a + 2 N_0 \delta_{ij} \delta_{kl} \kappa \pi_1^{ik} \pi_1^{jl} a + 4 N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_1^{jl} a + \\
& N_1 \pi_{\varphi 0} \pi_{\varphi 1} a^{(-3)} + 3 N_0 \pi_{\varphi 0} \pi_{\varphi 1} a^{(-3)} \psi_1 + \frac{3}{2} N_1 \pi_{\varphi 0}^2 a^{(-3)} \psi_1 + \frac{15}{4} N_0 \pi_{\varphi 0}^2 \psi_1^2 a^{(-3)} - \\
& 2 N_0 \delta^{ij} \partial_{ij} \psi_1 \kappa^{-1} \psi_1 a - 2 N_1 \delta^{ij} \partial_{ij} \psi_1 \kappa^{-1} a - 3 N_1 V_0 a^3 \psi_1 + \frac{3}{2} N_0 V_0 \psi_1^2 a^3 \\
& - 2 N_1 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_0^{jl} \psi_1 a - N_0 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_0^{jl} \psi_1^2 a + \frac{1}{2} N_0 \pi_{\varphi 1}^2 a^{(-3)} \\
& - 4 N_0 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_1^{jl} \psi_1 a - 3 N_0 \delta^{ij} \partial_i \psi_1 \partial_j \psi_1 \kappa^{-1} a.
\end{aligned} \tag{B.1}$$

Since,  $\varphi$  is unperturbed, variation of (B.1) with respect to  $\pi_{\varphi 1}$  vanishes.

$$\begin{aligned}
\frac{\delta \mathcal{H}_2^C}{\delta \pi_{\varphi 1}} &= 0 \\
\Rightarrow \pi_{\varphi 1} &= -\frac{N_1}{N_0} \pi_{\varphi 0} - 3 \pi_{\varphi 0} \psi_1
\end{aligned} \tag{B.2}$$

Explicit expression of  $\pi_1^{ij}$  is obtained by varying the above Hamiltonian with respect to  $\pi_1^{ij}$ .

$$\begin{aligned}
\partial_0 \gamma_{ij} &= \frac{\delta \mathcal{H}_2^C}{\delta \pi_1^{ij}} \\
\Rightarrow \pi_1^{ij} &= \kappa^{-1} \left( \frac{1}{2} N_0^{-1} a \delta^{ij} \partial_k N_1^k + 2 N_0^{-1} \delta^{ij} \partial_0 a \psi_1 + N_0^{-1} a \delta^{ij} \partial_0 \psi_1 - \frac{1}{2} N_0^{-1} a \delta^{kj} \partial_k N_1^i \right) \\
&\quad - N_0^{-1} N_1 \pi_0^{ij} + \pi_0^{ij} \psi_1
\end{aligned} \tag{B.3}$$

Using the above definitions and varying the Hamiltonian (B.1) with respect to  $N_1$ , we get the Hamiltonian Constraint, which, in conformal coordinate becomes

$$\begin{aligned}
\frac{\delta \mathcal{H}_2^C}{\delta N_1} &= 0 \\
\Rightarrow & -2 \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_1^{kl} a + \delta_{ij} \delta_{kl} \kappa \pi_0^{ij} \pi_0^{kl} \psi_1 a + 4 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_1^{jl} a - 2 \delta_{ij} \delta_{kl} \kappa \pi_0^{ik} \pi_0^{jl} \psi_1 a + \\
&\quad \pi_{\varphi 0} \pi_{\varphi 1} a^{(-3)} + \frac{3}{2} \pi_{\varphi 0}^2 a^{(-3)} \psi_1 - 2 \delta^{ij} \partial_{ij} \psi_1 \kappa^{-1} a - 3 V_0 a^3 \psi_1 = 0 \\
\Rightarrow & -3 H \psi_1' + \delta^{ij} \partial_{ij} \psi_1 - H \delta^{ij} \partial_{ij} B_1 - \kappa \phi_1 V_0 a^2 = 0.
\end{aligned} \tag{B.4}$$

Similarly, varying the Hamiltonian with respect to  $N_1^i$  leads to Momentum constraint and in conformal coordinate, it becomes

$$\begin{aligned}
\frac{\delta \mathcal{H}_2^C}{\delta N_1^i} &= 0 \\
\Rightarrow -2\delta_{ij}\partial_i\pi_1^{jk} + 4\delta_{ij}\partial_k\psi_1\pi_0^{ij} - 2\delta_{jk}\pi_0^{jk}\partial_i\psi_1 &= 0 \\
\Rightarrow \partial_i\psi_1' + H\partial_i\phi_1 &= 0.
\end{aligned} \tag{B.5}$$

Finally, the equation of motion of  $\psi_1$  is given by

$$\begin{aligned}
\delta^{ij} \left( \partial_0\pi_1^{ij} + \frac{\delta \mathcal{H}_2^C}{\delta \gamma_{ij}} \right) &= 0 \\
\Rightarrow \delta^{ij} \left( \partial_0\pi_1^{ij} + \frac{\delta \mathcal{H}_2^C}{\delta \psi_1} \frac{\partial \psi}{\partial \gamma_{ij}} \right) &= 0 \\
\Rightarrow 2\delta^{ij}H\partial_{ij}B_1 + \delta^{ij}\partial_{ij}B_1' + 6H\psi' + 3\psi'' + 3\kappa V_0\phi_1a^2 + \\
3H\phi_1' - \delta^{ij}\partial_{ij}\psi + \delta^{ij}\partial_{ij}\phi_1 &= 0.
\end{aligned} \tag{B.6}$$

It can be verified that above equations are consistent with Lagrangian equations of motion.

## C Perturbed Lagrangian equations of motion of Galilean scalar field in flat-slicing gauge

### C.1 Background equations

In flat-slicing gauge, using (4.23) and (4.24), (2.14) and (2.15), we get the zeroth order Lagrangian density of the Galilean field (5.1)

$$\mathcal{L}_{G0} = -3N_0^{-1}\kappa^{-1}a'^2a - 2a'N_0^{(-3)}\varphi_0'^3a^2 - N_0V_0a^3. \tag{C.1}$$

Varying the Lagrangian (C.1) with respect to the zeroth order Lapse function  $N_0$ , we get Hamiltonian constraint at zeroth order and it is given by

$$3N_0^{-2}\kappa^{-1}a'^2a + 6N_0^{-4}\varphi_0'^3a^2a' - V_0a^3 = 0. \tag{C.2}$$

In conformal coordinate, the above equation takes the form

$$V_0a^2 - 6Ha^{-2}\varphi_0'^3 - 3\kappa^{-1}H^2 = 0. \tag{C.3}$$

Similarly, equation of motion of  $a$  in conformal coordinate is given by

$$6H\varphi_0'^3a^{(-2)} - 6\varphi_0''\varphi_0'^2a^{-2} + 3\kappa^{-1}H^2 - 6\frac{a''}{a}\kappa^{-1} + 3V_0a^2 = 0 \tag{C.4}$$

and equation of motion of  $\varphi_0$  in conformal coordinate is given by

$$\frac{a''}{2aH}\varphi_0'^2 - \frac{1}{2}H\varphi_0'^2 + \varphi_0''\varphi_0' - \frac{1}{12H}V_\varphi a^4 = 0. \tag{C.5}$$

## C.2 First order perturbation

In conformal coordinate, it can be shown that, the equation of motion of  $N_1$  or the first order Hamiltonian constraint is

$$24 H \phi_1 \varphi_0'^3 a^{(-2)} - 18 \varphi_1' H \varphi_0'^2 a^{(-2)} + V_\varphi a^2 \varphi_1 + 2 \delta^{ij} \partial_{ij} B_1 \varphi_0'^3 a^{-2} + 2 \delta^{ij} \partial_{ij} \varphi_1 \varphi_0'^2 a^{-2} + 2 \delta^{ij} H \partial_{ij} B_1 \kappa^{-1} + 6 \phi_1 \kappa^{-1} H = 0. \quad (C.6)$$

Similarly, equation of motion of  $N_1^i$  or the first order perturbed Momentum constraint is the following

$$-6 H \partial_i \varphi_1 \varphi_0'^2 - 2 \partial_i \phi_1 \varphi_0'^3 + 2 \partial_i \varphi_1' \varphi_0'^2 - 2 H \partial_i \phi_1 \kappa^{-1} a^2 = 0 \quad (C.7)$$

and equation of motion of  $\varphi_1$  is given by

$$\begin{aligned} & -18 \phi_1 \varphi_0'^2 H^2 + 12 \varphi_0' \varphi_1' H^2 + 18 \phi_1' H \varphi_0'^2 \\ & + 36 \varphi_0' H \varphi_0'' \phi_1 - 12 \varphi_0' H \varphi_1'' - 12 \varphi_1' H \varphi_0'' \\ & - 12 \varphi_0' \varphi_1' a'' a^{-1} + 6 \delta^{ij} H \partial_{ij} B_1 \varphi_0'^2 + 4 \delta^{ij} \varphi_0' \varphi_0'' \partial_{ij} B_1 + \\ & 4 \delta^{ij} \varphi_0' H \partial_{ij} \varphi_1 + 4 \delta^{ij} \varphi_0'' \partial_{ij} \varphi_1 + 2 \delta^{ij} \partial_{ij} B_1' \varphi_0'^2 + V_\varphi \phi_1 a^4 + \\ & V_{\varphi\varphi} a^4 \varphi_1 + 2 \delta^{ij} \partial_{ij} \phi_1 \varphi_0'^2 + 18 a'' \phi_1 \varphi_0'^2 a^{-1} = 0. \end{aligned} \quad (C.8)$$

## D Interaction Hamiltonian for higher order correlations of Galilean scalar field

Third order Interaction Hamiltonian of Galilean scalar field model (5.1), which is needed to compute Bi-spectrum, can be obtained by substituting (5.42) in the Hamiltonian (5.16) and extract the third order perturbed part as

$$\begin{aligned} \mathcal{H}_3 = & \frac{1}{2} N_1 V_{\varphi\varphi} \varphi_1'^2 a^3 + N_1^i \pi_{\lambda 1} \partial_i \lambda_1 + N_1^i \pi_{\varphi 1} \partial_i \varphi_1 - \pi_{\lambda 0} \pi_{\varphi 1} N_1'^2 - \pi_{\lambda 1} \pi_{\varphi 0} N_1'^2 \\ & - 2 N_0 N_1 \pi_{\lambda 1} \pi_{\varphi 1} + \pi_{\lambda 1} \lambda_1 \partial_i N_1^i - 3 N_1 S_0 N_0'^2 \pi_{\lambda 1}^2 a^3 - 6 N_0 \pi_{\lambda 0} \pi_{\lambda 1} S_0 N_1'^2 a^3 \\ & - S_0 N_1'^3 \pi_{\lambda 0}^2 a^3 - S_1 N_0'^3 \pi_{\lambda 1}^2 a^3 - 6 N_1 \pi_{\lambda 0} \pi_{\lambda 1} S_1 N_0'^2 a^3 - 3 N_0 S_1 N_1'^2 \pi_{\lambda 0}^2 a^3 \\ & - \frac{3}{2} N_0 \pi_{\lambda 0} \pi_{\lambda 1} \kappa \lambda_1'^2 a^{(-3)} - \frac{3}{2} N_0 \kappa \lambda_0 \lambda_1 \pi_{\lambda 1}^2 a^{(-3)} - \frac{3}{4} N_1 \kappa \pi_{\lambda 0}^2 \lambda_1'^2 a^{(-3)} - \\ & N_1^i \pi_{\lambda 1} \lambda_0 \partial_i N_1 N_0^{-1} - N_1^i \pi_{\lambda 0} \lambda_1 \partial_i N_1 N_0^{-1} - N_1 \delta^{ij} \lambda_0 \partial_i N_1 \partial_j \varphi_1 N_0^{(-2)} a^{(-2)} + \\ & \delta^{ij} \lambda_1 \partial_i N_1 \partial_j \varphi_1 N_0^{-1} a^{(-2)} + N_1 S_0 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a + N_0 S_1 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a - \\ & N_1 \pi_{\lambda 1} \delta_{ij} \kappa \lambda_0 \pi_1^{ij} a^{-1} - N_0 \pi_{\lambda 1} \delta_{ij} \kappa \lambda_1 \pi_1^{ij} a^{-1} - N_1 \pi_{\lambda 0} \delta_{ij} \kappa \lambda_1 \pi_1^{ij} a^{-1} - \\ & N_1 \delta_{ij} \delta_{kl} \kappa \pi_1^{ij} \pi_1^{kl} a + 2 N_1 \delta_{ij} \delta_{kl} \kappa \pi_1^{ik} \pi_1^{jl} a - \frac{3}{4} N_1 \kappa \pi_{\lambda 1}^2 \lambda_0'^2 a^{(-3)} + \\ & N_1 N_1^i \pi_{\lambda 0} \lambda_0 \partial_i N_1 N_0^{(-2)} - N_1 \pi_{\lambda 1} \delta_{ij} \kappa \lambda_1 \pi_0^{ij} a^{-1} - 3 N_1 \pi_{\lambda 0} \pi_{\lambda 1} \kappa \lambda_0 \lambda_1 a^{(-3)}. \end{aligned} \quad (D.1)$$

First order and zeroth order Hamiltonian relations along with (4.25) can be used to express the third order Hamiltonian in terms of a single field and its derivatives so that we can compute the correlation function.

Similarly, fourth Order interaction Hamiltonian, which helps to compute Tri-spectrum, is given by

$$\begin{aligned}
\mathcal{H}_4 = & -\pi_{\lambda 1} \pi_{\varphi 1} N_1^2 - 3 N_0 S_0 N_1^2 \pi_{\lambda 1}^2 a^3 - 2 \pi_{\lambda 0} \pi_{\lambda 1} S_0 N_1^3 a^3 - \\
& S_1 N_1^3 \pi_{\lambda 0}^2 a^3 - \frac{3}{4} N_0 \kappa \pi_{\lambda 1}^2 \lambda_1^2 a^{(-3)} - \frac{3}{2} N_1 \pi_{\lambda 0} \pi_{\lambda 1} \kappa \lambda_1^2 a^{(-3)} - \\
& N_1^i \pi_{\lambda 0} \lambda_0 \partial_i N_1 N_0^{(-3)} N_1^2 + N_1 N_1^i \pi_{\lambda 1} \lambda_0 \partial_i N_1 N_0^{(-2)} - \\
& N_1^i \pi_{\lambda 1} \lambda_1 \partial_i N_1 N_0^{-1} + \delta^{ij} \lambda_0 \partial_i N_1 \partial_j \varphi_1 N_0^{(-3)} N_1^2 a^{(-2)} + \\
& N_1 S_1 \delta^{ij} \partial_i \varphi_1 \partial_j \varphi_1 a - N_1 \pi_{\lambda 1} \delta_{ij} \kappa \lambda_1 \pi_1^{ij} a^{-1} - 6 N_0 \pi_{\lambda 0} \pi_{\lambda 1} S_1 N_1^2 a^3 \\
& - 3 N_1 S_1 N_0^2 \pi_{\lambda 1}^2 a^3 + N_1 N_1^i \pi_{\lambda 0} \lambda_1 \partial_i N_1 N_0^{(-2)} - \frac{3}{2} N_1 \kappa \lambda_0 \lambda_1 \pi_{\lambda 1}^2 a^{(-3)} \\
& - N_1 \delta^{ij} \lambda_1 \partial_i N_1 \partial_j \varphi_1 N_0^{(-2)} a^{(-2)}. \tag{D.2}
\end{aligned}$$

## E Galilean and Canonical scalar field

Now we proceed to the action where both canonical part is present in a Galilean fields model. The action is given by

$$\begin{aligned}
\mathcal{S} = & \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa} R - \frac{1}{2} \beta \partial_\mu \varphi \partial_\nu \varphi - \alpha g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \square \varphi - V(\varphi) \right] \\
= & \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa} R - \frac{1}{2} \beta \partial_\mu \varphi \partial_\nu \varphi - \alpha g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi S - V(\varphi) \right] + \int d^4 x \lambda (S - \square \varphi). \tag{E.1}
\end{aligned}$$

$$= \int \mathcal{L} d^4 x \tag{E.2}$$

Expanding the above action using ADM decomposition, we get

$$\begin{aligned}
\mathcal{L} = & S\lambda - NV\gamma^{\frac{1}{2}} + \gamma^{ij} \partial_i \lambda \partial_j \varphi + \lambda \partial_i \gamma^{ij} \partial_j \varphi - \lambda' \varphi' N^{(-2)} + \frac{1}{2} N^{(3)} R \gamma^{\frac{1}{2}} \kappa^{-1} + N^i \lambda' \partial_i \varphi N^{(-2)} + \\
& N^i \varphi' \partial_i \lambda N^{(-2)} + \frac{1}{2} \beta N^{-1} \gamma^{\frac{1}{2}} \varphi'^2 + \lambda N' \varphi' N^{(-3)} - N^i N^j \partial_i \lambda \partial_j \varphi N^{(-2)} - N^i \lambda N' \partial_i \varphi N^{(-3)} - \\
& N^i \lambda \varphi' \partial_i N N^{(-3)} + S\alpha N^{-1} \gamma^{\frac{1}{2}} \varphi'^2 + \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{jk} \partial_l \varphi - \frac{1}{2} \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{kl} \partial_j \varphi + \\
& \frac{1}{2} \gamma^{ij} \lambda \gamma_{ij}' \varphi' N^{(-2)} - \gamma^{ij} \lambda \partial_i N \partial_j \varphi N^{-1} - \frac{1}{2} N \beta \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} + N^i N^j \lambda \partial_i N \partial_j \varphi N^{(-3)} - \\
& N^i \beta \varphi' \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}} - \frac{1}{2} N^i \gamma^{jk} \lambda \gamma_{jk}' \partial_i \varphi N^{(-2)} - \frac{1}{2} N^i \gamma^{jk} \lambda \varphi' \partial_i \gamma_{jk} N^{(-2)} - \\
& \frac{1}{2} K^{ij} K^{kl} N \gamma_{ij} \gamma_{kl} \gamma^{\frac{1}{2}} \kappa^{-1} + \frac{1}{2} K^{ij} K^{kl} N \gamma_{ik} \gamma_{jl} \gamma^{\frac{1}{2}} \kappa^{-1} - N S \alpha \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} + \\
& \frac{1}{2} N^i N^j \beta \partial_i \varphi \partial_j \varphi N^{-1} \gamma^{\frac{1}{2}} + \frac{1}{2} N^i N^j \gamma^{kl} \lambda \partial_i \gamma_{kl} \partial_j \varphi N^{(-2)} - 2 N^i S \alpha \partial_0 \varphi \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}} + \\
& N^i N^j S \alpha \partial_i \varphi \partial_j \varphi N^{-1} \gamma^{\frac{1}{2}} \tag{E.3}
\end{aligned}$$

Definition of all momenta are same as we derived in the Galilean case except  $\pi_\varphi$  and it is given by

$$\begin{aligned}\pi_\varphi = & -\lambda' N^{(-2)} + N^i \partial_i \lambda N^{(-2)} + \beta N^{-1} \gamma^{\frac{1}{2}} \varphi' + \lambda N' N^{(-3)} - N^i \lambda \partial_i N N^{(-3)} + \\ & 2 S \alpha N^{-1} \gamma^{\frac{1}{2}} \varphi' + \frac{1}{2} \gamma^{ij} \lambda \gamma_{ij}' N^{(-2)} - N^i \beta \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}} - \frac{1}{2} N^i \gamma^{jk} \lambda \partial_i \gamma_{jk} N^{(-2)} - \\ & 2 N^i S \alpha \partial_i \varphi N^{-1} \gamma^{\frac{1}{2}}.\end{aligned}\tag{E.4}$$

Then the Dirac-Hamiltonian becomes,

$$\begin{aligned}\mathcal{H}_D = & -S\lambda + NV\gamma^{\frac{1}{2}} + N^i \pi_\lambda \partial_i \lambda + N^i \pi_\varphi \partial_i \varphi - \pi_\lambda \pi_\varphi N^2 + \pi_\lambda \lambda \partial_i N^i + 2\gamma_{ij} \partial_k N^i \pi^{jk} - \\ & \gamma^{ij} \partial_i \lambda \partial_j \varphi - \lambda \partial_i \gamma^{ij} \partial_j \varphi - \frac{1}{2} N^{(3)} R \gamma^{\frac{1}{2}} \kappa^{-1} - \frac{1}{2} \beta N^3 \pi_\lambda^2 \gamma^{\frac{1}{2}} - \frac{3}{4} N \kappa \pi_\lambda^2 \gamma^{-\frac{1}{2}} \lambda^2 - \\ & N^i \pi_\lambda \lambda \partial_i N N^{-1} + N^i \partial_i \gamma_{lm} \pi^{lm} - N^i \partial_l \gamma_{im} \pi^{lm} + N^i \partial_m \gamma_{il} \pi^{lm} - S \alpha N^3 \pi_\lambda^2 \gamma^{\frac{1}{2}} - \\ & \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{jk} \partial_l \varphi + \frac{1}{2} \gamma^{ij} \gamma^{kl} \lambda \partial_i \gamma_{kl} \partial_j \varphi + \gamma^{ij} \lambda \partial_i N \partial_j \varphi N^{-1} + \frac{1}{2} N \beta \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} - \\ & N \pi_\lambda \gamma_{ij} \kappa \lambda \pi^{ij} \gamma^{-\frac{1}{2}} + N S \alpha \gamma^{ij} \partial_i \varphi \partial_j \varphi \gamma^{\frac{1}{2}} - N \gamma_{ij} \gamma_{kl} \kappa \pi^{ij} \pi^{kl} \gamma^{-\frac{1}{2}} + 2 N \gamma_{ij} \gamma_{kl} \kappa \pi^{ik} \pi^{jl} \gamma^{-\frac{1}{2}} \\ & + \xi(\pi_N + \lambda N^{-1} \pi_\lambda)\end{aligned}\tag{E.5}$$

### E.1 Zeroth order

Zeroth order Hamiltonian, in terms of  $\pi_a$ , takes the form

$$\begin{aligned}\mathcal{H}_{D0} = & -S_0 \lambda_0 + N_0 V a^3 - \pi_{\lambda 0} \pi_{\varphi 0} N_0^2 - \frac{1}{2} \beta N_0^3 \pi_{\lambda 0}^2 a^3 - \frac{3}{4} N_0 \kappa \pi_{\lambda 0}^2 \lambda_0^2 a^{(-3)} \\ & - S_0 \alpha N_0^3 \pi_{\lambda 0}^2 a^3 - \frac{1}{2} N_0 \pi_a \pi_{\lambda 0} \kappa \lambda_0 a^{(-2)} - \frac{1}{12} N_0 \kappa \pi_a^2 a^{-1} + \xi_0 (\pi_{N0} + \pi_{\lambda 0} \lambda_0 N_0^{-1}).\end{aligned}\tag{E.6}$$

Zeroth order Hamiltonian constraint or the equation of motion of  $N_0$  is given by

$$V a^2 + \frac{1}{2} \beta \varphi_0'^2 - 3 \kappa^{-1} H^2 - 6 \alpha H a^{(-2)} \varphi_0'^3 = 0.\tag{E.7}$$

Similarly, equations of motion of  $a$  and  $\varphi_0$  are given by

$$\begin{aligned}6 \alpha H \varphi_0'^3 a^{(-2)} - 6 \alpha \varphi_0'' \varphi_0'^2 a^{-2} + 3 \kappa^{-1} H^2 - \\ 6 a'' a^{-1} \kappa^{-1} + 3 V a^2 - \frac{3}{2} \beta \varphi_0'^2 = 0,\end{aligned}\tag{E.8}$$

$$\begin{aligned}V_\varphi a^2 - 6 \alpha a'' \varphi_0'^2 a^{(-3)} + 6 \alpha \varphi_0'^2 H^2 a^{(-2)} - 12 \alpha \varphi_0' H \varphi_0'' a^{(-2)} \\ + 2 \beta \varphi_0' H + \beta \varphi_0'' = 0.\end{aligned}\tag{E.9}$$

## E.2 First order

first order Hamilton's equations are obtained from second order perturbed Hamiltonian and it is given by

$$\begin{aligned}
\mathcal{H}_{D2} = & -S_1\lambda_1 + N_1V_\varphi a^3\varphi_1 + \frac{1}{2}N_0V_{\varphi\varphi}\varphi_1^2a^3 + N_1^i\pi_{\lambda 0}\partial_i\lambda_1 + N_1^i\pi_{\varphi 0}\partial_i\varphi_1 - \\
& 2N_0N_1\pi_{\lambda 0}\pi_{\varphi 1} - 2N_0N_1\pi_{\lambda 1}\pi_{\varphi 0} - \pi_{\lambda 1}\pi_{\varphi 1}N_0^2 + \pi_{\lambda 1}\lambda_0\partial_iN_1^i - \\
& \delta^{ij}\partial_i\lambda_1\partial_j\varphi_1a^{(-2)} - \frac{1}{2}\beta N_0^3\pi_{\lambda 1}^2a^3 - 3N_1\pi_{\lambda 0}\pi_{\lambda 1}\beta N_0^2a^3 - \frac{3}{2}N_0\beta N_1^2\pi_{\lambda 0}^2a^3 - \\
& \frac{3}{4}N_0\kappa\pi_{\lambda 0}^2\lambda_1^2a^{(-3)} - 3N_0\pi_{\lambda 0}\pi_{\lambda 1}\kappa\lambda_0\lambda_1a^{(-3)} - \frac{3}{4}N_0\kappa\pi_{\lambda 1}^2\lambda_0^2a^{(-3)} - \\
& \frac{3}{2}N_1\pi_{\lambda 0}\pi_{\lambda 1}\kappa\lambda_0^2a^{(-3)} - N_1^i\pi_{\lambda 0}\lambda_0\partial_iN_1N_0^{-1} - S_0\alpha N_0^3\pi_{\lambda 1}^2a^3 - \\
& 3N_0S_0\alpha N_1^2\pi_{\lambda 0}^2a^3 - 2\pi_{\lambda 0}\pi_{\lambda 1}S_1\alpha N_0^3a^3 - 3N_1S_1\alpha N_0^2\pi_{\lambda 0}^2a^3 + \\
& \frac{1}{2}N_0\beta\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a - N_0\pi_{\lambda 1}\delta_{ij}\kappa\lambda_0\pi_1^{ij}a^{-1} - N_1\pi_{\lambda 0}\delta_{ij}\kappa\lambda_0\pi_1^{ij}a^{-1} - \\
& N_0\pi_{\lambda 0}\delta_{ij}\kappa\lambda_1\pi_1^{ij}a^{-1} - N_0\pi_{\lambda 1}\delta_{ij}\kappa\lambda_1\pi_0^{ij}a^{-1} - N_1\pi_{\lambda 0}\delta_{ij}\kappa\lambda_1\pi_0^{ij}a^{-1} - \\
& N_0\delta_{ij}\delta_{kl}\kappa\pi_1^{ij}\pi_1^{kl}a - 2N_1\delta_{ij}\delta_{kl}\kappa\pi_0^{ij}\pi_1^{kl}a + 2N_0\delta_{ij}\delta_{kl}\kappa\pi_1^{ik}\pi_1^{jl}a + \\
& \pi_{N1}\xi_1 - N_1\pi_{\lambda 0}\lambda_0N_0^{(-2)}\xi_1 + \pi_{\lambda 0}\lambda_0N_0^{(-3)}N_1^2\xi_0 + \pi_{\lambda 1}\lambda_0N_0^{-1}\xi_1 + \\
& \pi_{\lambda 0}\lambda_1N_0^{-1}\xi_1 - N_1\pi_{\lambda 0}\lambda_1N_0^{(-2)}\xi_0 + \pi_{\lambda 1}\lambda_1N_0^{-1}\xi_0 - \pi_{\lambda 0}\pi_{\varphi 0}N_1^2 + \\
& \pi_{\lambda 0}\lambda_1\partial_iN_1^i + 2\delta_{ij}\partial_kN_1^i\pi_1^{jk}a^2 - \frac{3}{2}N_1\kappa\lambda_0\lambda_1\pi_{\lambda 0}^2a^{(-3)} - \\
& 6N_1\pi_{\lambda 0}\pi_{\lambda 1}S_0\alpha N_0^2a^3 + \delta^{ij}\lambda_0\partial_iN_1\partial_j\varphi_1N_0^{-1}a^{(-2)} - N_1\pi_{\lambda 1}\delta_{ij}\kappa\lambda_0\pi_0^{ij}a^{-1} + \\
& N_0S_0\alpha\delta^{ij}\partial_i\varphi_1\partial_j\varphi_1a - N_1\pi_{\lambda 1}\lambda_0N_0^{(-2)}\xi_0 + 4N_1\delta_{ij}\delta_{kl}\kappa\pi_0^{ik}\pi_1^{jl}a
\end{aligned} \tag{E.10}$$

Since we have got the perturbed second order Hamiltonian, we can obtained the field equations using Hamilton's equations. First order Momentum constraint or the equation of motion of  $N_1^i$  is given by

$$\begin{aligned}
& -6\alpha H\partial_i\varphi_1\varphi_0'^2 + \beta\varphi_0'\partial_i\varphi_1a^2 - 2\alpha\partial_i\phi_1\varphi_0'^3 + 2\alpha\partial_i\varphi_1'\varphi_0'^2 - \\
& 2H\partial_i\phi_1\kappa^{-1}a^2 = 0.
\end{aligned} \tag{E.11}$$

Similarly, First order Hamiltonian constraint or the equation of motion of  $N_1$  is given by

$$\begin{aligned}
& 24\alpha H\phi_1\varphi_0'^3a^{(-2)} - 18\alpha\varphi_1'H\varphi_0'^2a^{(-2)} + V_\varphi a^2\varphi_1 - \beta\phi_1\varphi_0'^2 + \\
& 2\alpha\delta^{ij}\partial_{ij}B_1\varphi_0'^3a^{-2} + \beta\varphi_0'\varphi_1' + 2\alpha\delta^{ij}\partial_{ij}\varphi_1\varphi_0'^2a^{-2} \\
& + 2\delta^{ij}H\partial_{ij}B_1\kappa^{-1} + 6\phi_1\kappa^{-1}H = 0
\end{aligned} \tag{E.12}$$

and the equation of motion of  $\varphi_1$  is given by

$$\begin{aligned}
& -18\alpha\phi_1\varphi_0'^2H^2 + 12\alpha\varphi_0'\varphi_1'H^2 + 18\alpha\phi_1'H\varphi_0'^2 \\
& + 36\alpha\varphi_0'H\varphi_0''\phi_1 - 12\alpha\varphi_0'H\varphi_1'' - 12\alpha\varphi_1'H\varphi_0'' \\
& + 18\alpha a''\phi_1\varphi_0'^2a^{-1} - 12\alpha\varphi_0'\varphi_1'a''a^{-1} - 2\beta\varphi_0'H\phi_1a^2 - \beta\phi_1'\varphi_0'a^2 \\
& - \beta\varphi_0''\phi_1a^2 + 6\alpha\delta^{ij}H\partial_{ij}B_1\varphi_0'^2 + 4\alpha\delta^{ij}\varphi_0'\varphi_0''\partial_{ij}B_1 + 2\beta\varphi_1'H a^2 \\
& + \beta\varphi_1''a^2 + 4\alpha\delta^{ij}\varphi_0'H\partial_{ij}\varphi_1 + 4\alpha\delta^{ij}\varphi_0''\partial_{ij}\varphi_1 + 2\alpha\delta^{ij}\partial_{ij}B_1'\varphi_0'^2 \\
& + V_\varphi\phi_1a^4 + V_{\varphi\varphi}a^4\varphi_1 - \beta\delta^{ij}\varphi_0'\partial_{ij}B_1a^2 + 2\alpha\delta^{ij}\partial_{ij}\phi_1\varphi_0'^2 - \beta\delta^{ij}\partial_{ij}\varphi_1a^2 = 0.
\end{aligned} \tag{E.13}$$



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